Nora Brambilla (TU Munich)

QQQ Potential at N^2LO

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based on

N. Brambilla, J. Ghiglieri, A. Vairo

The Three-quark static potential in perturbation theory

Phys.Rev.D81:054031,2010. e-Print: arXiv:0911.3541 [hep-ph]

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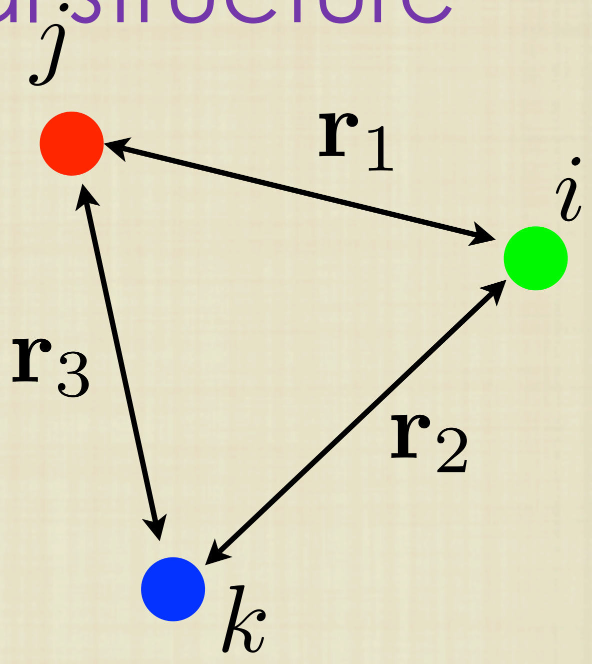

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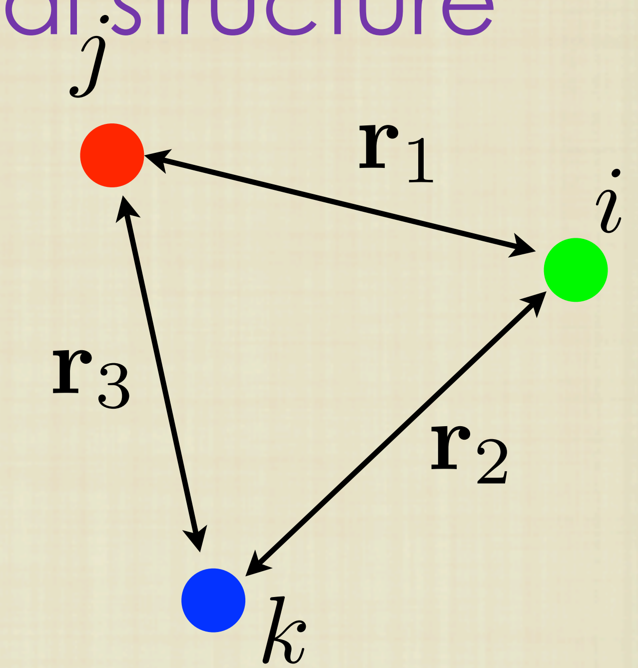

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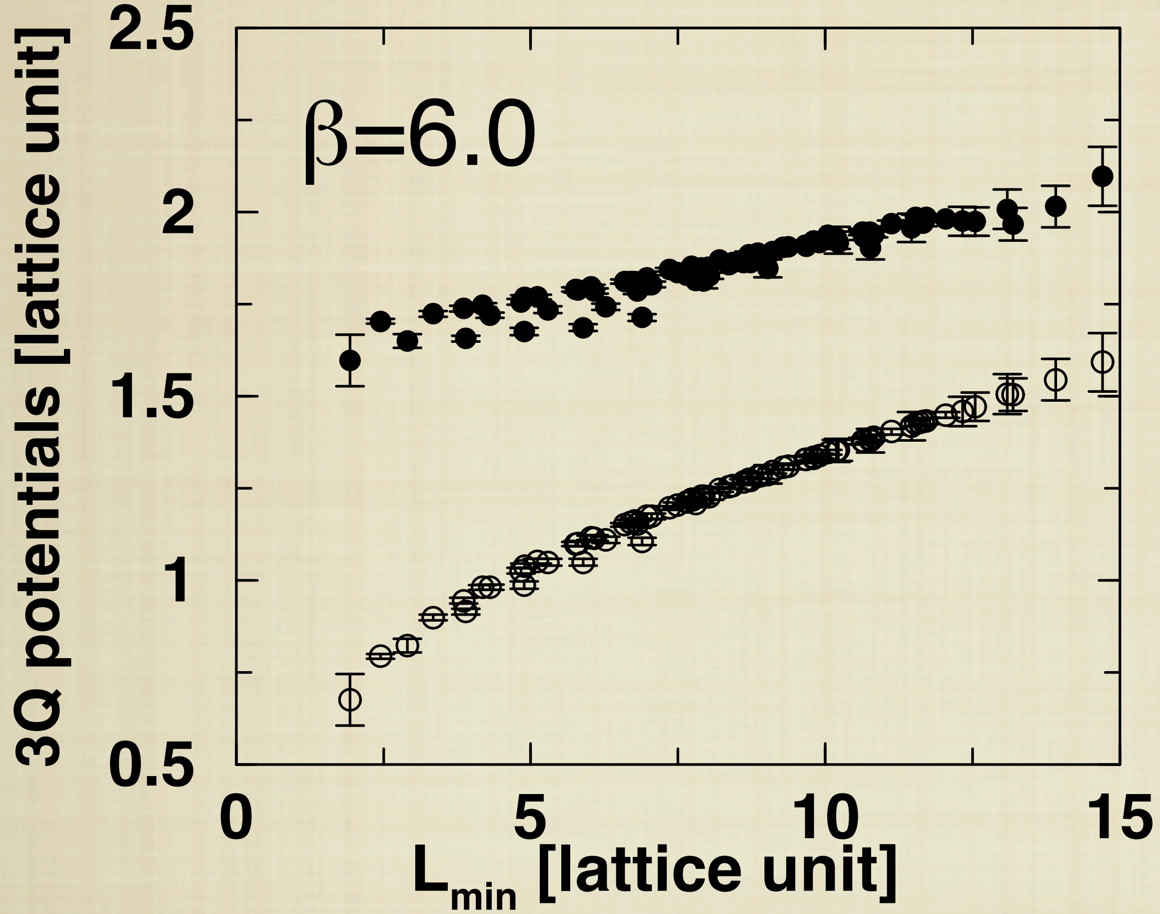
in perturbation theory the tree level has been known in all color channels, e.g. for the singlet

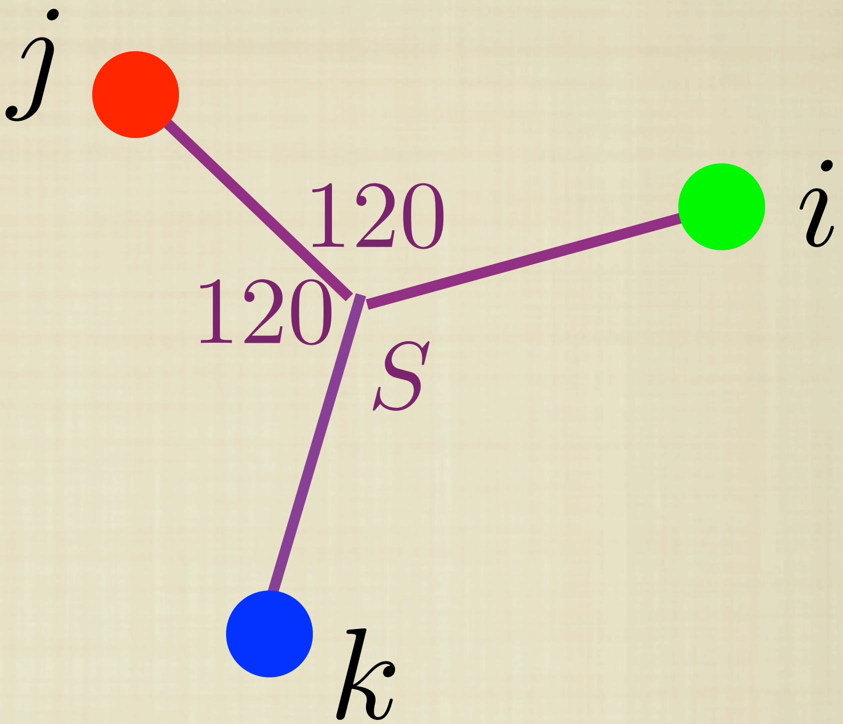
$$V_s(\mathfrak{r}) = -\frac{2}{3}\alpha_s \left(\frac{1}{|\mathbf{r}_1|} + \frac{1}{|\mathbf{r}_2|} + \frac{1}{|\mathbf{r}_3|} \right)$$

The QQQ potential is calculated on the lattice in the singlet channel with a particular interest in the large distance



Takahashi Suganuma **PRD70** (2002)

$a = 0.1\mathrm{fm}$



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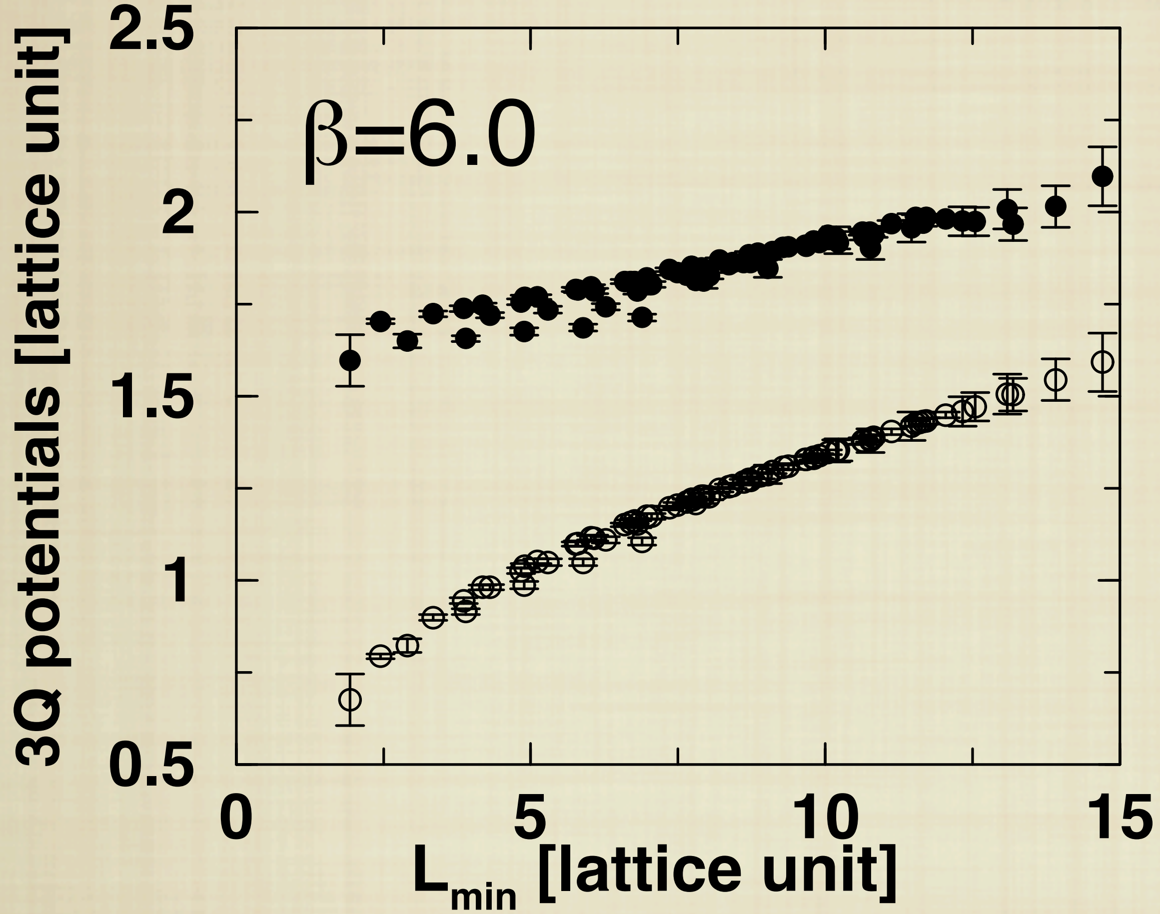

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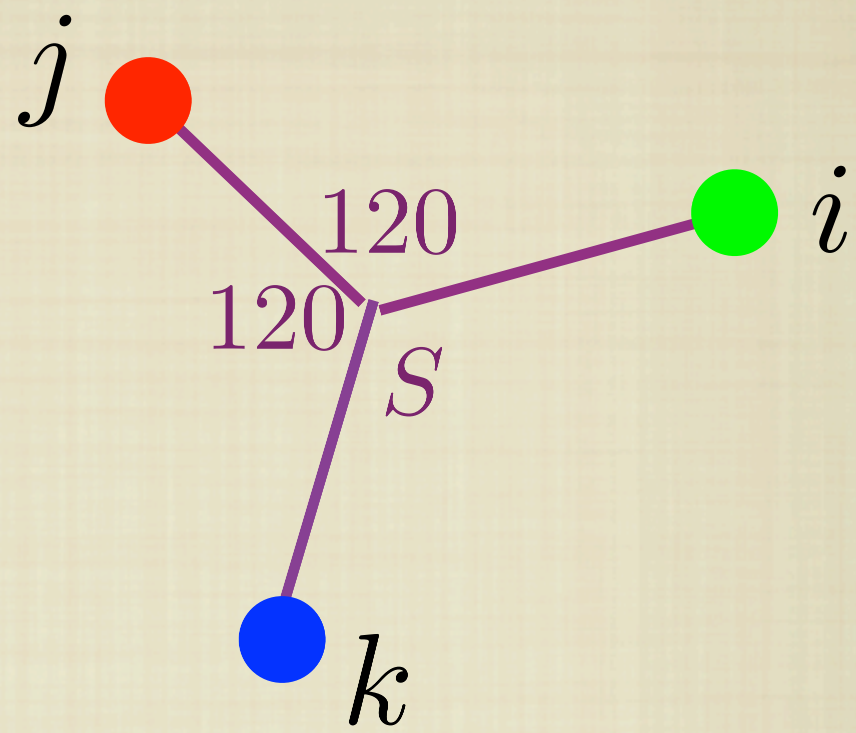
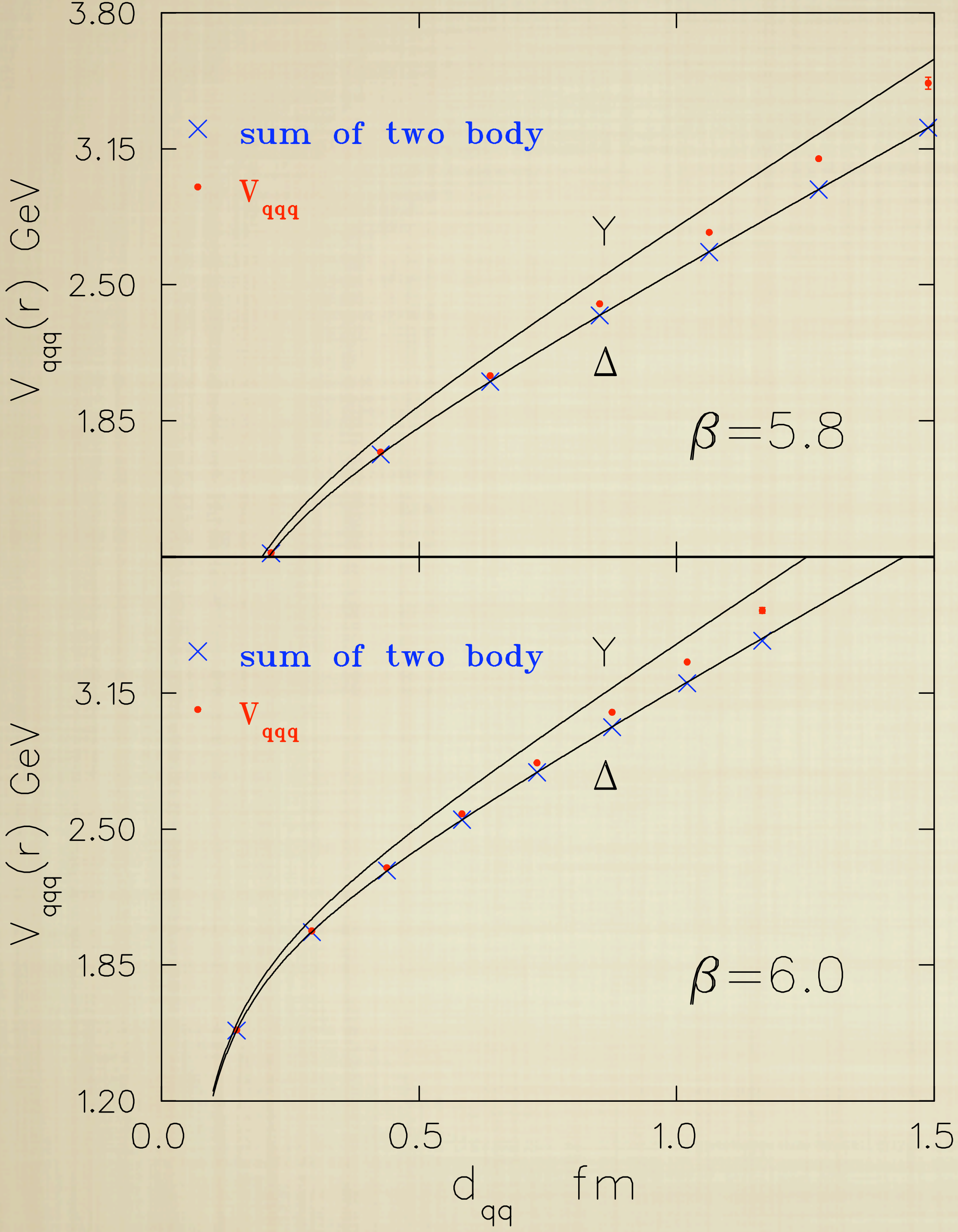


$$L_{\min} = \overline{jS} + \overline{iS} + \overline{kS}$$

area law: 3 flux tubes joining in one point-> three body forces

The precise behaviour of the QQQ potential is still object of investigation on the lattice



hep-lat/0209062
equilateral geometry,
d_qq =qq distance

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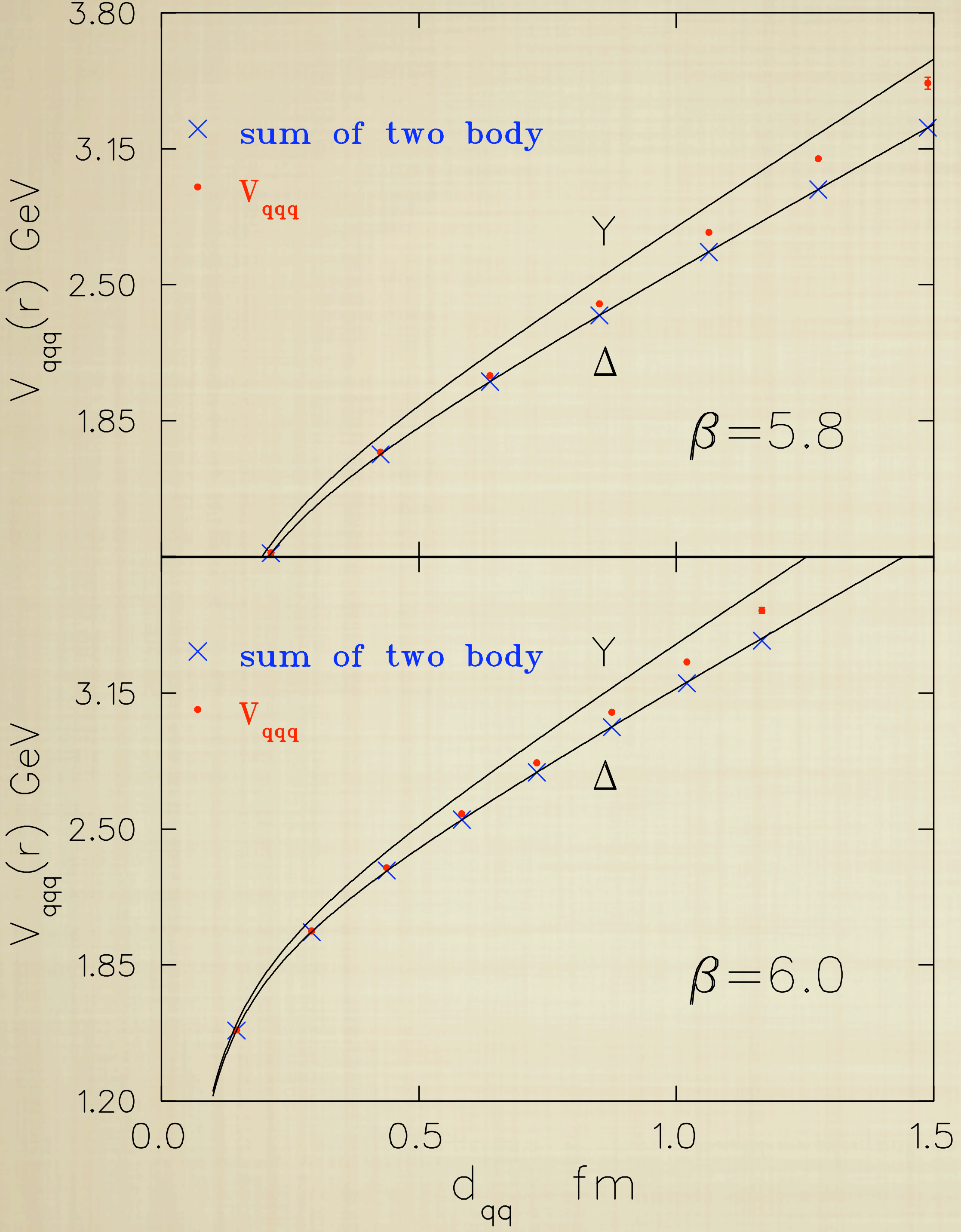

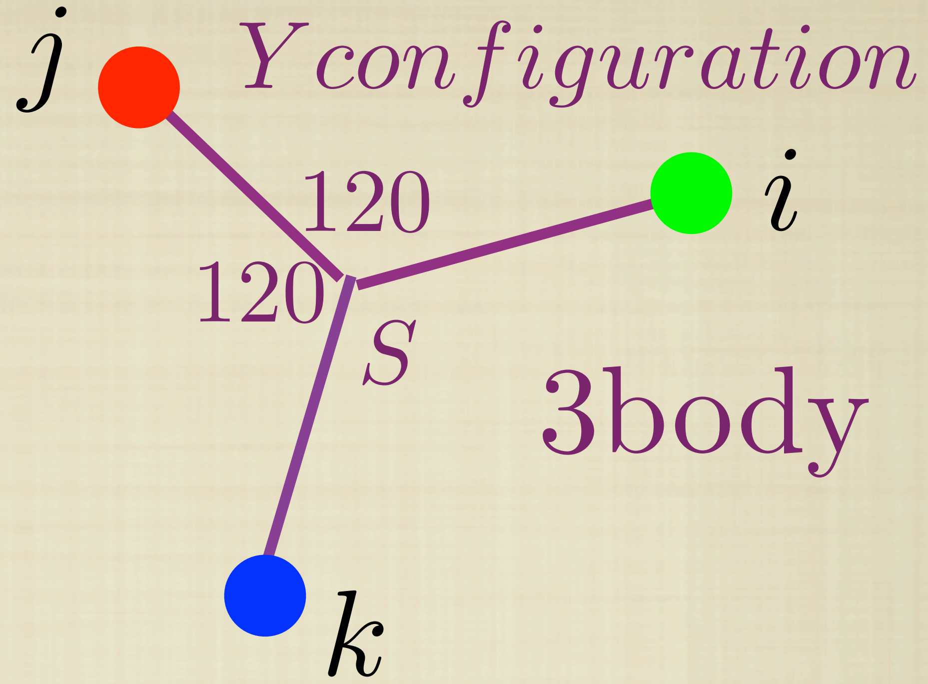

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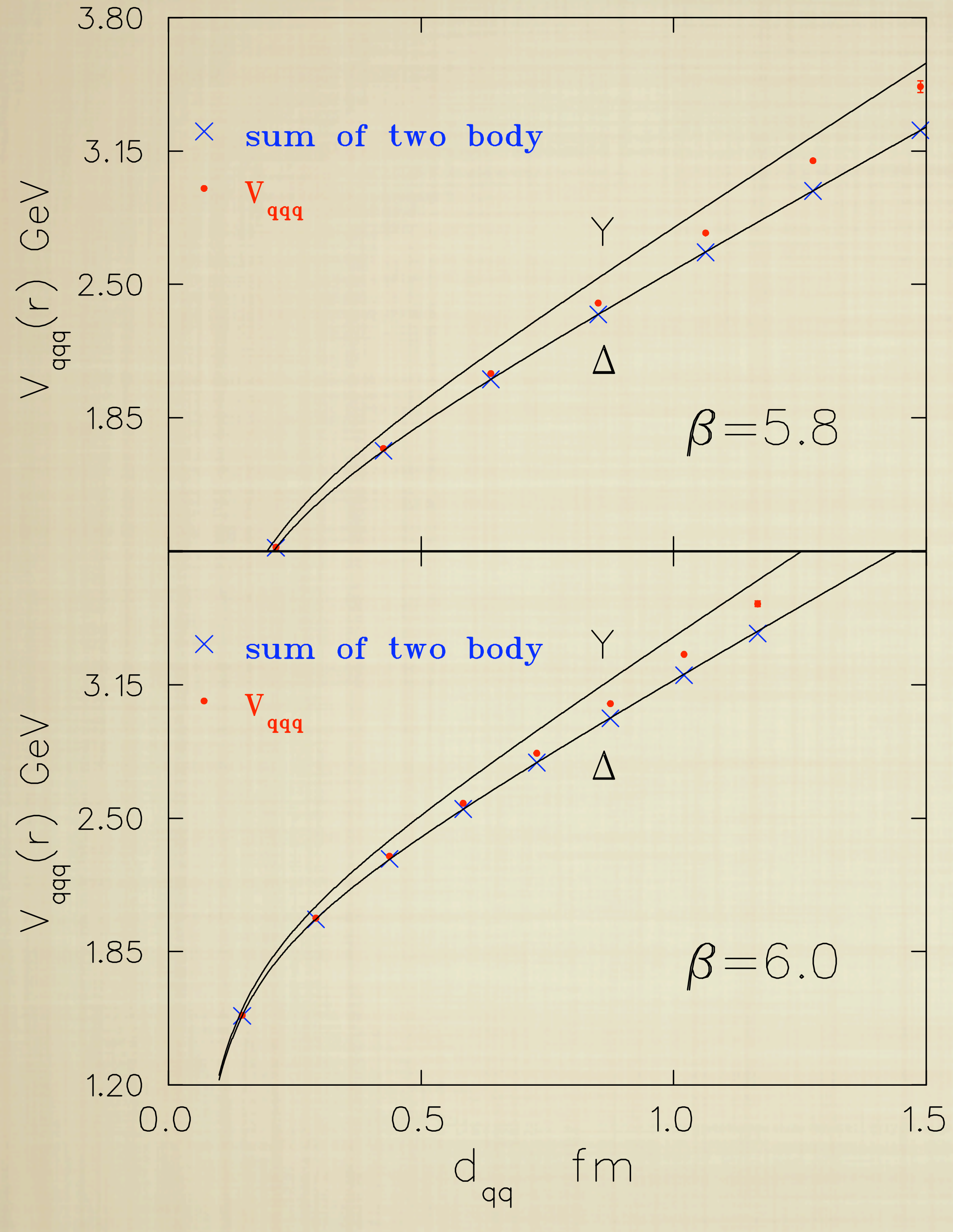

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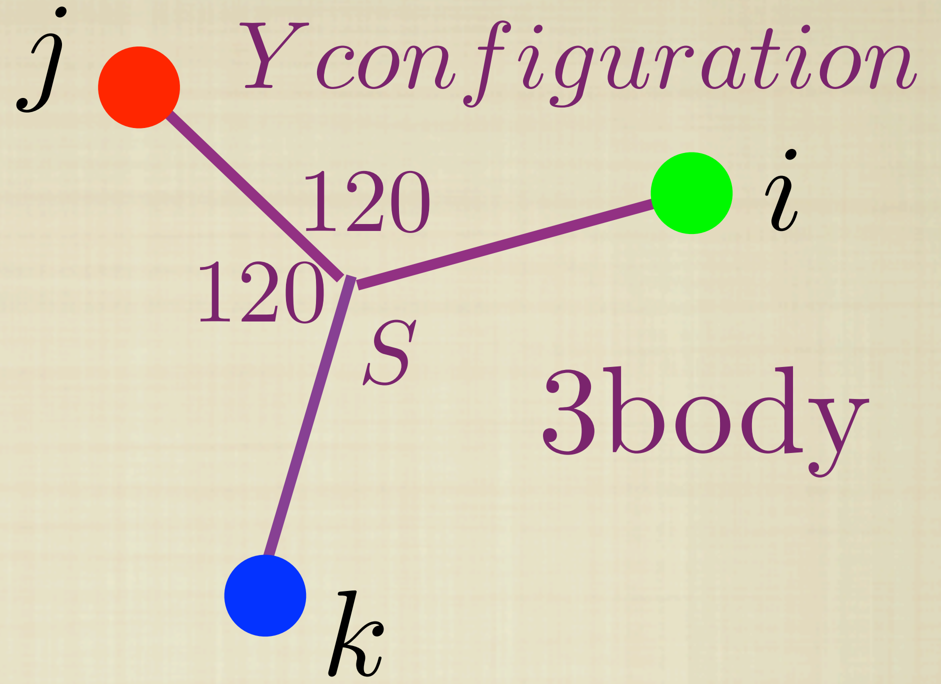

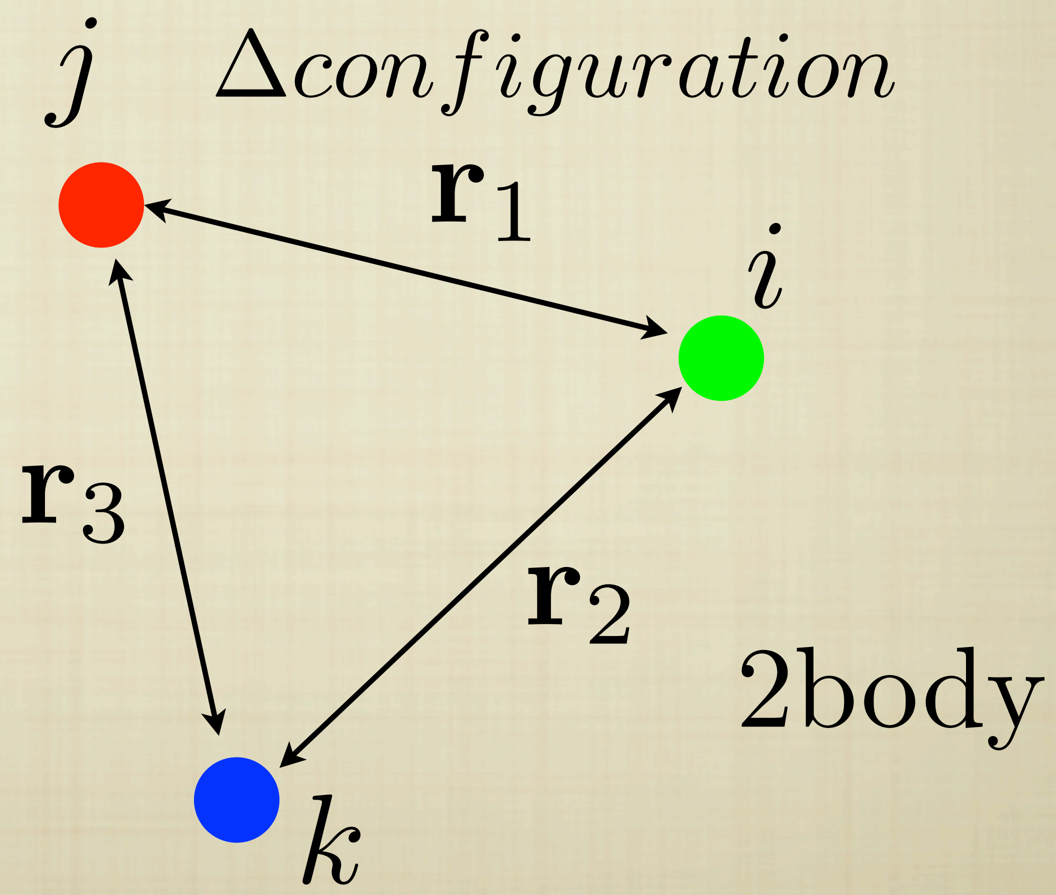

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	Bjorken [4]	This work	Vijande *et al* [24]
Ω_{bcc}	8.200 ± 0.090	7.98 ± 0.07	–
Ω_{ccc}	4.925 ± 0.090	4.76 ± 0.06	4.632
Ω_{bbb}	14.760 ± 0.180	14.37 ± 0.08	–
Ω_{bbc}	11.480 ± 0.120	11.19 ± 0.08	–

Yu Jia, hep-ph/0607290 with tree level perturbative potential

The QQQ richer color structure can become particularly interesting at finite temperature

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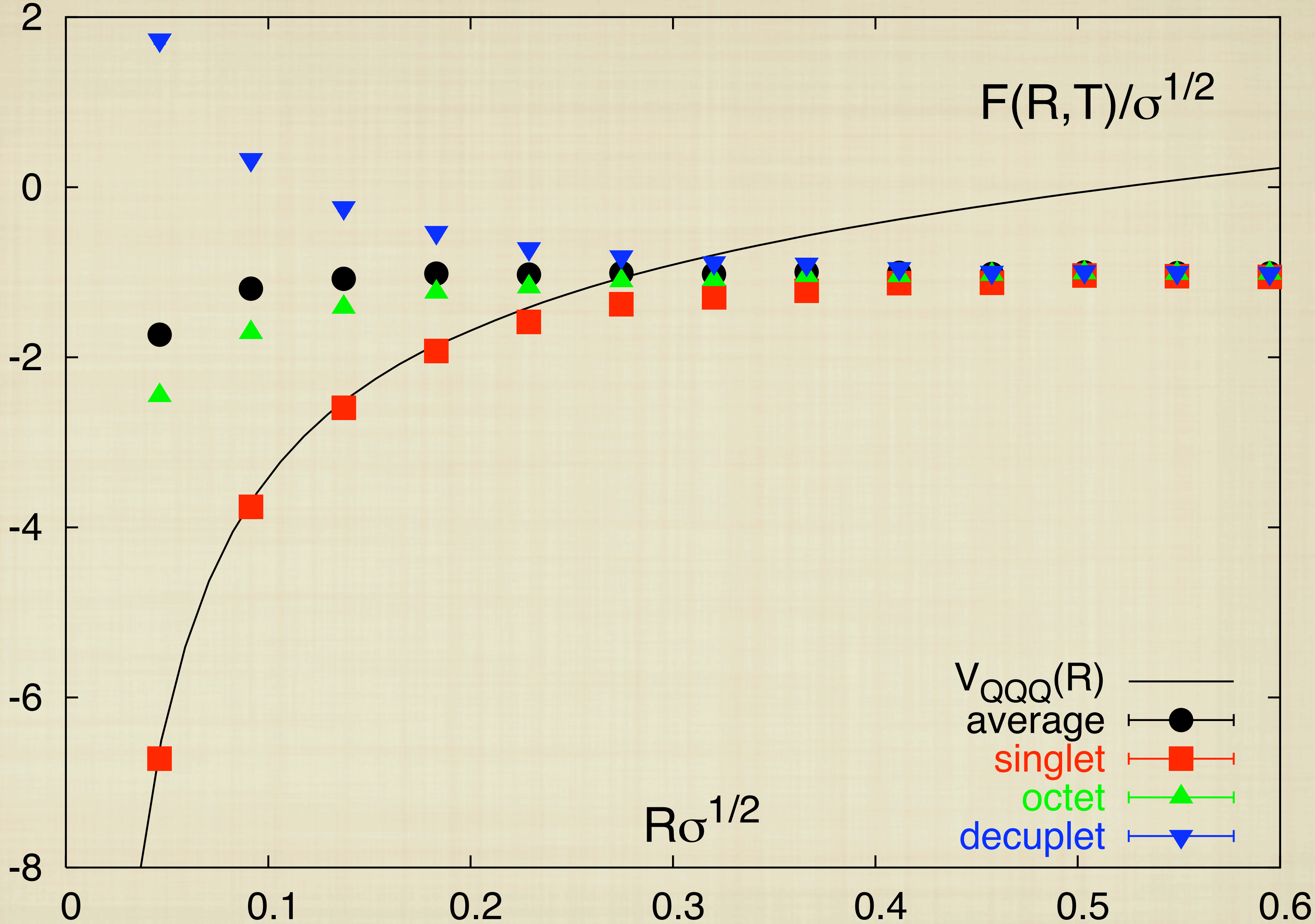
lattice calculation of the free energy at finite T



$$T = 6T_C$$

Hübner Kaczmarek Karsch Vogt **PRD77** (2008)

The Calculation of the QQQ at N^2LO

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- *The (weakly coupled) EFT for QQQ baryons contains:*
  q, *gluons*, $(QQQ)_1 = S$, $(QQQ)_8 = (O^{A1}, \ldots, O^{A8})$, $(QQQ)_8 = (O^{S1}, \ldots, O^{S8})$ *and* $(QQQ)_{10} = (\Delta^1, \ldots, \Delta^{10})$.

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$$
\begin{aligned}
\mathcal{L}_{\rm pNRQCD} \quad = \quad & \int d^3\mathbf{r}\, d^3\mathbf{r}' \,\Big\{ S^\dagger \left[i\partial_0 - V_S^{(0)} \right] S + O^\dagger \left[iD_0 - V_O^{(0)} \right] O \\
& + \Delta^\dagger \left[iD_0 - V_\Delta^{(0)} \right] \Delta + \mathcal{O}\left(\frac{1}{m}, r, r' \right) \Big\} + \mathcal{L}_{\rm gauge} + \mathcal{L}_{\rm l.q.}
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$V_S \quad V_O^A \quad V_O^S \quad V_\Delta$ Wilson coefficients to be calculated in the matching

Matching the QQQ potential

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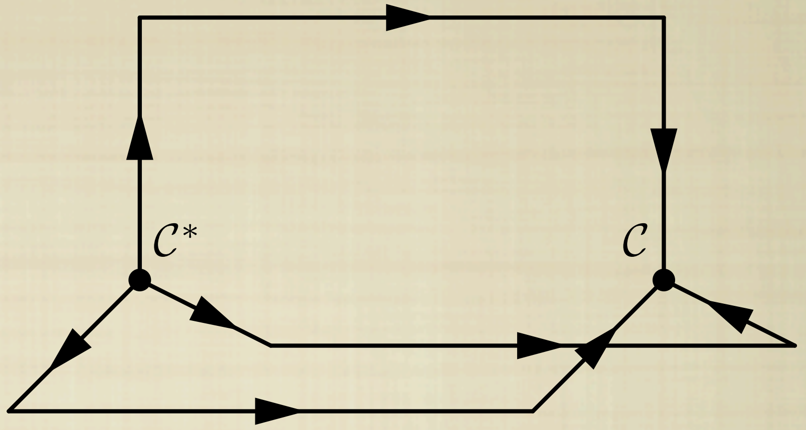
UP TO TWO LOOPS:

$$\mathfrak{r} = \mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3$$

$$V_{\mathcal{C}}(\mathfrak{r}) = \lim_{T_W \to \infty} \frac{i}{T_W} \ln \frac{\langle 0 | \mathcal{C}^u \, W \, \mathcal{C}^{v\dagger} | 0 \rangle}{\mathcal{C}^u_{mno} \mathcal{C}^{v\dagger}_{mno}},$$



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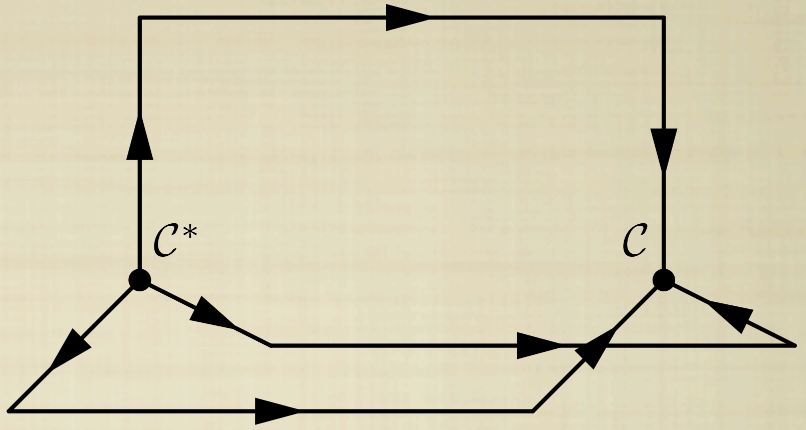

$$\frac{\langle 0 | \mathcal{C}^u \, W \, \mathcal{C}^{v\dagger} | 0 \rangle}{\mathcal{C}^u_{mno} \mathcal{C}^{v\dagger}_{mno}} = 1 + \mathcal{M}^{(0)}(\mathcal{C}, \mathfrak{r}) + \mathcal{M}^{(1)}(\mathcal{C}, \mathfrak{r}) + \mathcal{M}^{(2)}(\mathcal{C}, \mathfrak{r}) + \ldots\,,$$

$(n) -> g^{2n+2}$ or α_s^{n+1}

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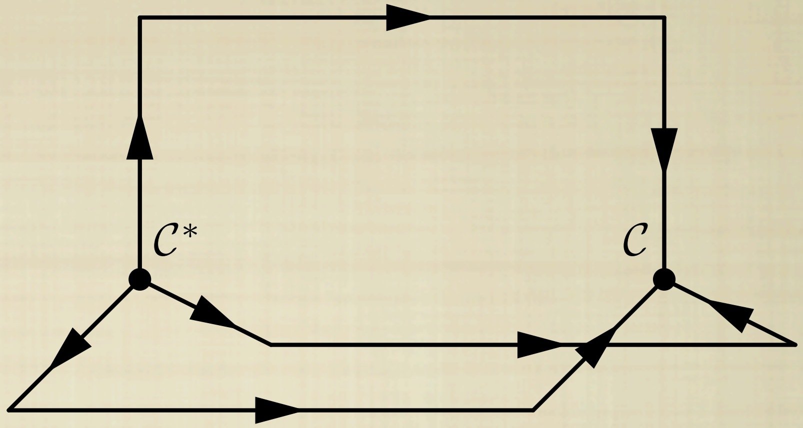

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$$V_{\mathcal{C}}(\mathfrak{r}) = V_{\mathcal{C}}^{(0)}(\mathfrak{r}) + V_{\mathcal{C}}^{(1)}(\mathfrak{r}) + V_{\mathcal{C}}^{(2)}(\mathfrak{r}) + \dots\,,$$

the potential is reconstructed throught the "potential exponentiation"

$$\begin{aligned}
V_{\mathcal{C}}^{(0)}(\mathfrak{r}) &= \lim_{T_W\to\infty} \frac{i}{T_W}\mathcal{M}^{(0)}(\mathcal{C},\mathfrak{r}), \\
V_{\mathcal{C}}^{(1)}(\mathfrak{r}) &= \lim_{T_W\to\infty} \frac{i}{T_W}\left(\mathcal{M}^{(1)}(\mathcal{C},\mathfrak{r}) - \frac{1}{2}\mathcal{M}^{(0)\,2}(\mathcal{C},\mathfrak{r})\right), \\
V_{\mathcal{C}}^{(2)}(\mathfrak{r}) &= \lim_{T_W\to\infty} \frac{i}{T_W}\left(\mathcal{M}^{(2)}(\mathcal{C},\mathfrak{r}) - \mathcal{M}^{(0)}(\mathcal{C},\mathfrak{r})\mathcal{M}^{(1)}(\mathcal{C},\mathfrak{r}) + \frac{1}{3}\mathcal{M}^{(0)\,3}(\mathcal{C},\mathfrak{r})\right) \\
\dots & \qquad \dots
\end{aligned}$$

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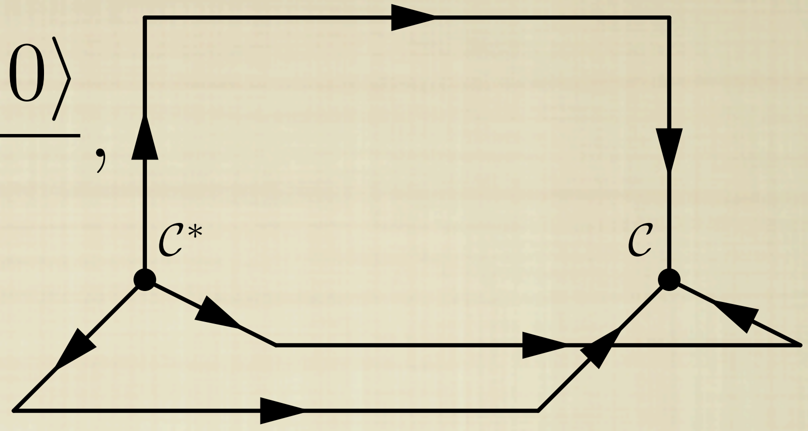

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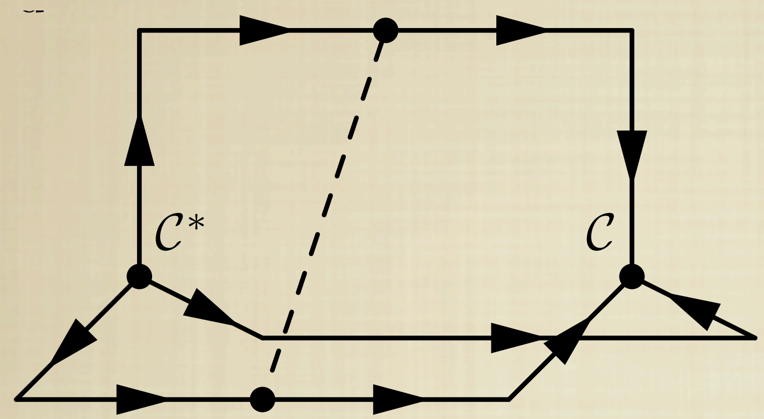
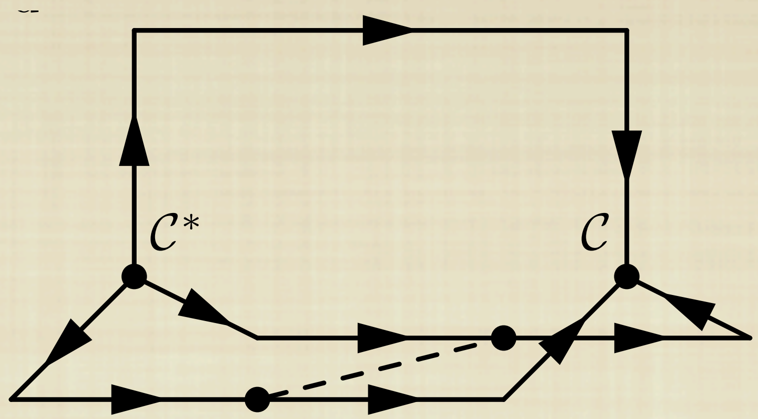
$$V_{\mathcal{C}}(\mathfrak{r}) = V_{\mathcal{C}}^{(0)}(\mathfrak{r}) + V_{\mathcal{C}}^{(1)}(\mathfrak{r}) + V_{\mathcal{C}}^{(2)}(\mathfrak{r}) + \dots,$$

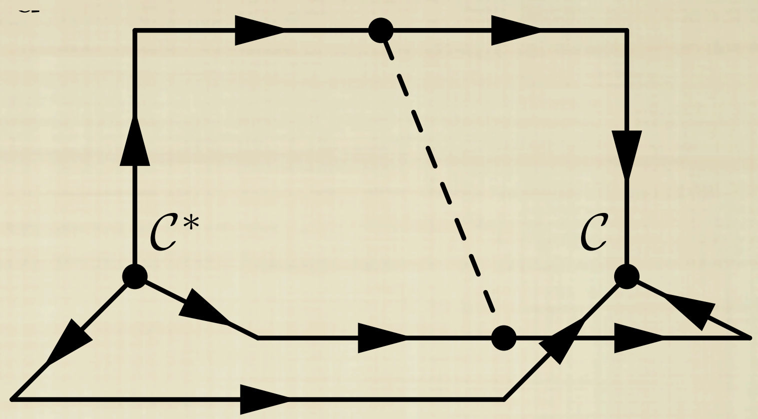
the potential is a sum of two-and three-body contributions

$$V(\mathfrak{r}) = \sum_{q=1}^{3} V_2(\mathbf{r}_q) + V_3(\mathfrak{r})$$

QQQ potential at LO







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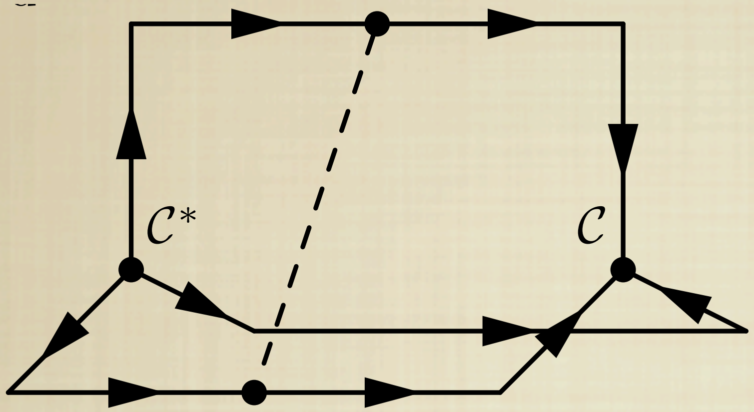

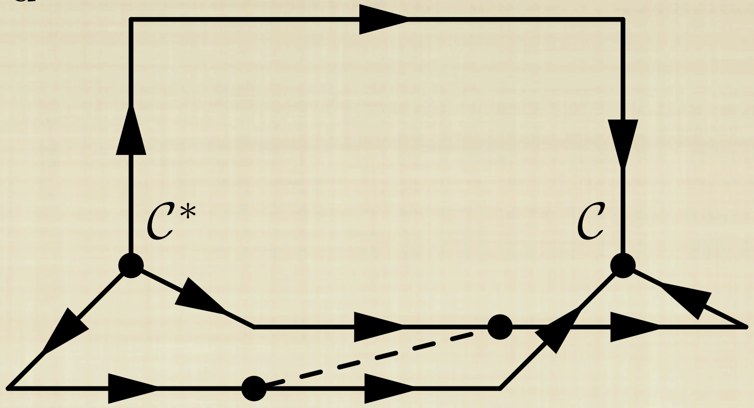

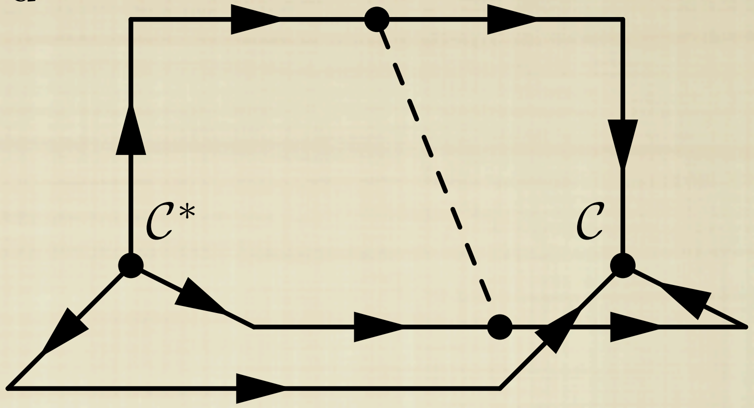

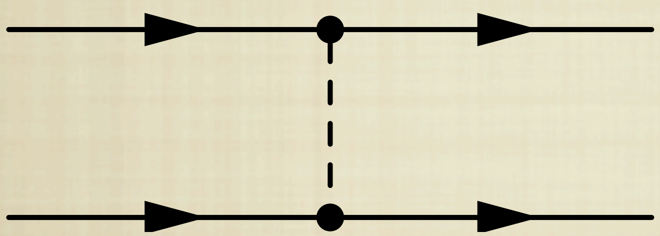

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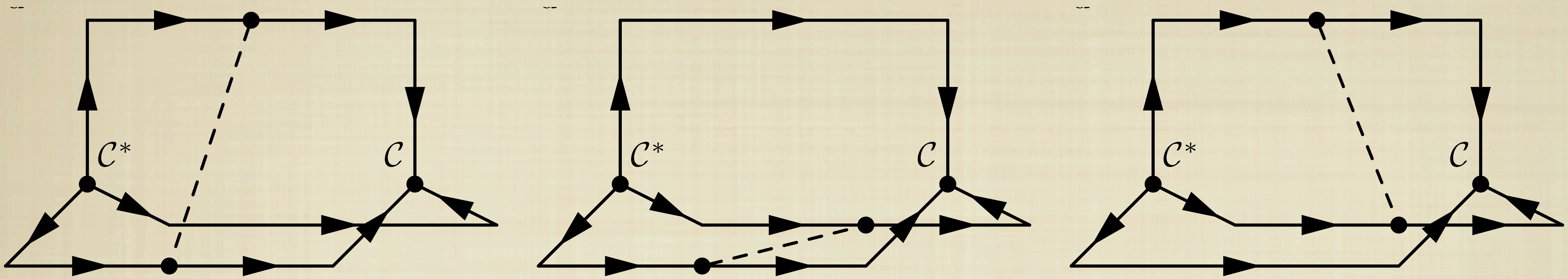
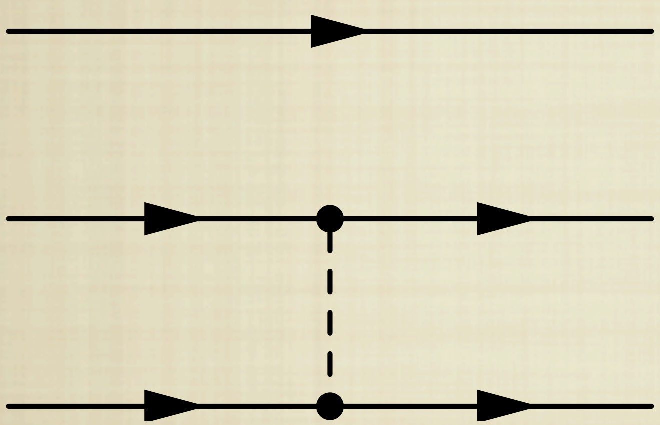
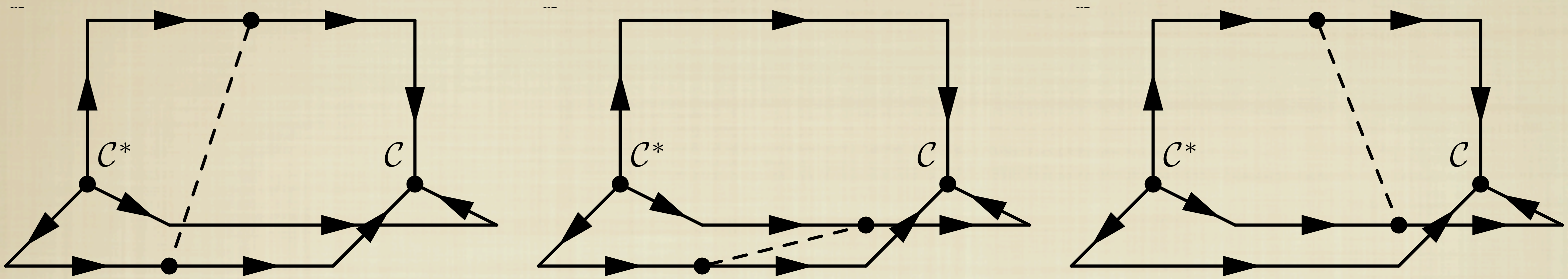
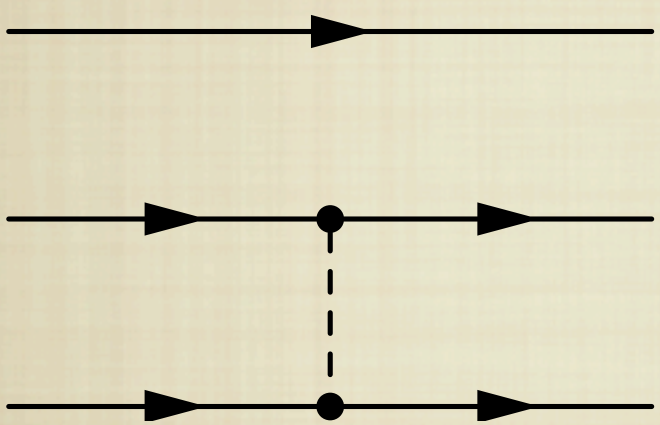




$$V^0_{\mathcal{C}}(\mathfrak{r}) = \sum_{q=1}^{3} f^0_q(\mathcal{C}) \frac{\alpha_s}{|\mathbf{r}_q|}$$

QQQ potential at LO





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color factor

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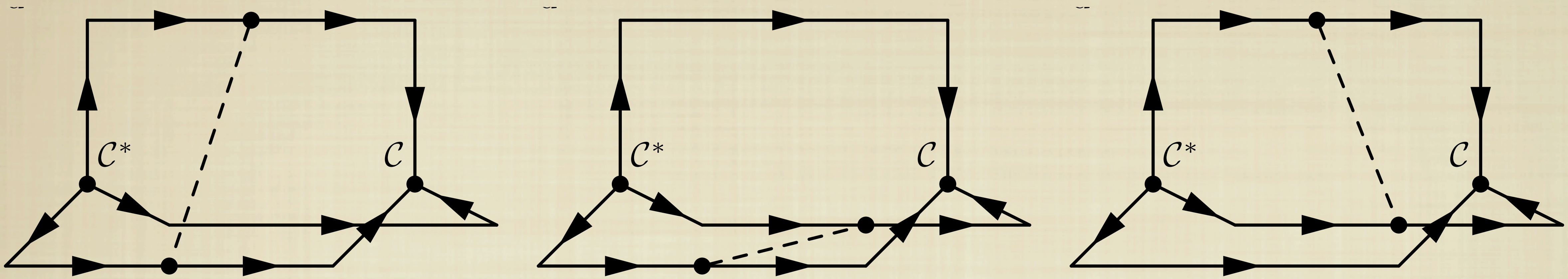

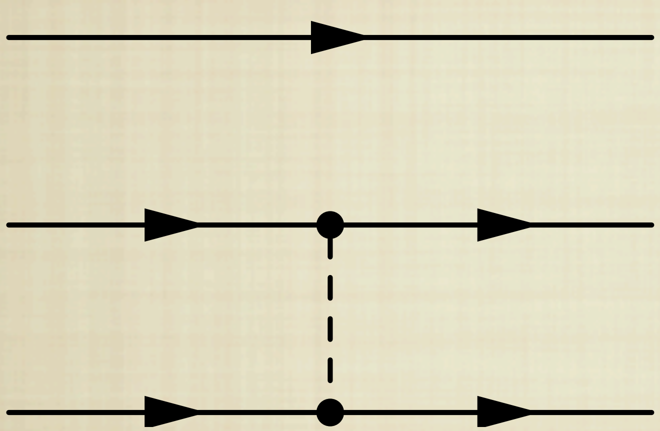

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color factor

perturbative
diagram calculation

Singlet and decuplet potential at LO

- The computation of the color factors yields

$$V_s(\mathfrak{r}) = -\frac{2}{3}\alpha_s\left(\frac{1}{|\mathbf{r}_1|}+\frac{1}{|\mathbf{r}_2|}+\frac{1}{|\mathbf{r}_3|}\right) \quad V_d(\mathfrak{r}) = \frac{1}{3}\alpha_s\left(\frac{1}{|\mathbf{r}_1|}+\frac{1}{|\mathbf{r}_2|}+\frac{1}{|\mathbf{r}_3|}\right)$$

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$$V_s(\mathfrak{r}) = -\underbrace{\frac{2}{3}}_{\text{Antitriplet diquark (QQ) color factor (antisymmetric)}}\alpha_s\left(\frac{1}{|\mathbf{r}_1|}+\frac{1}{|\mathbf{r}_2|}+\frac{1}{|\mathbf{r}_3|}\right) \quad V_d(\mathfrak{r}) = \frac{1}{3}\alpha_s\left(\frac{1}{|\mathbf{r}_1|}+\frac{1}{|\mathbf{r}_2|}+\frac{1}{|\mathbf{r}_3|}\right)$$

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Antitriplet diquark (QQ)
color factor
(antisymmetric)

Sextet diquark (QQ)
color factor
(symmetric)

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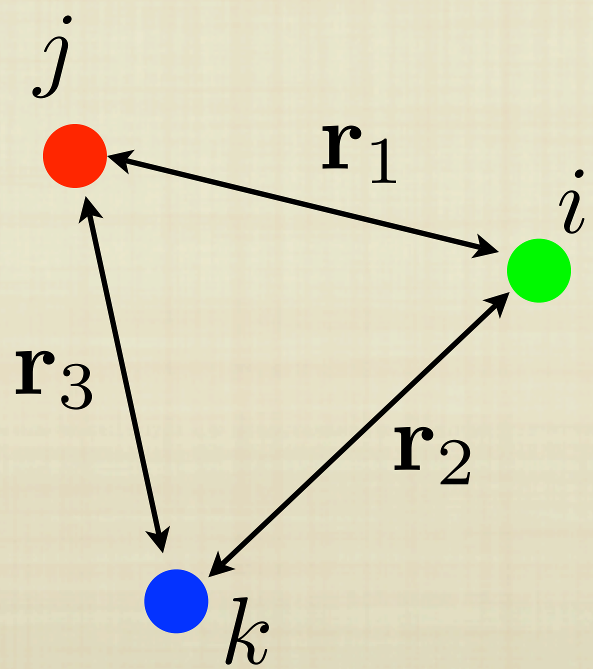
Antitriplet diquark (QQ) color factor (antisymmetric)



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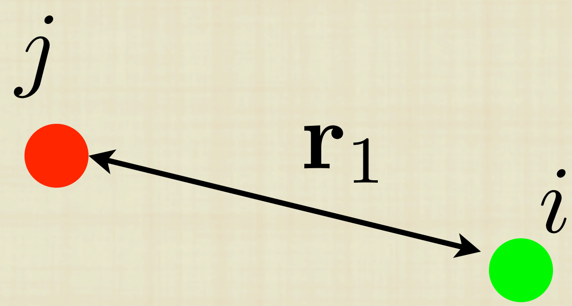
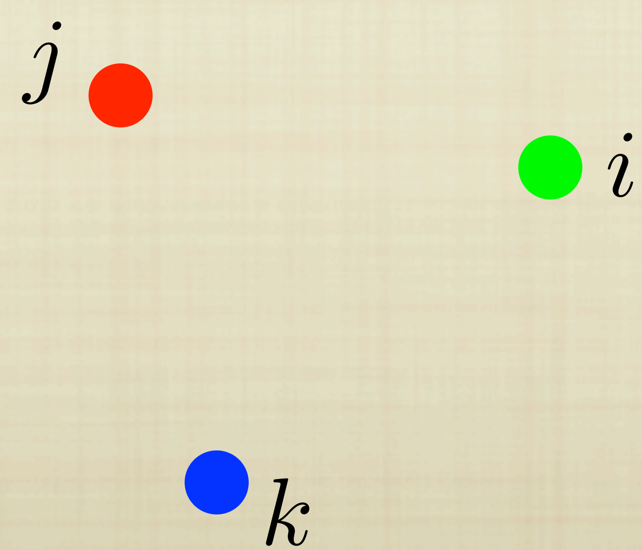


Sextet diquark (QQ)
color factor
(symmetric)

The octets at LO

- The simple one-gluon-exchange is enough to mix the two representations, i.e. nonzero color amplitude from O^A to O^S
- Potential has then to be defined as a matrix



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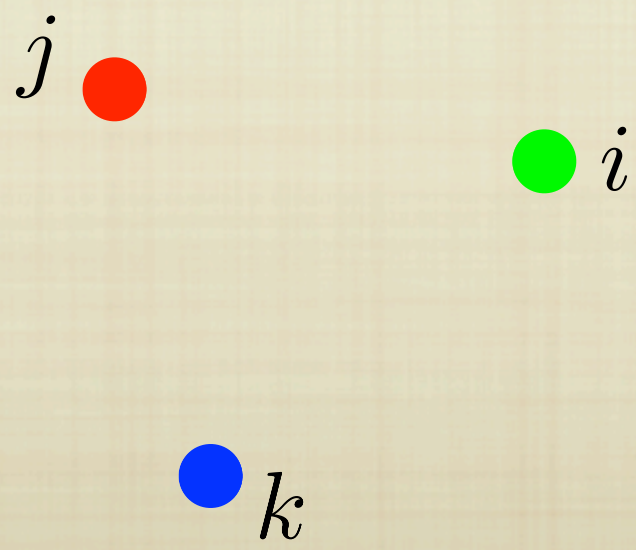

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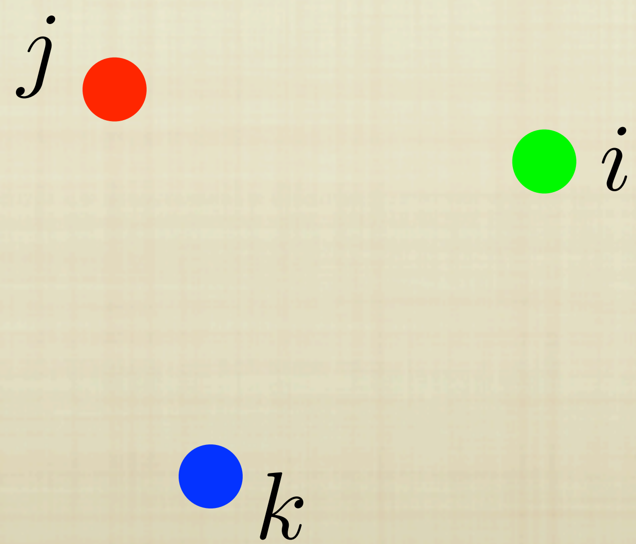

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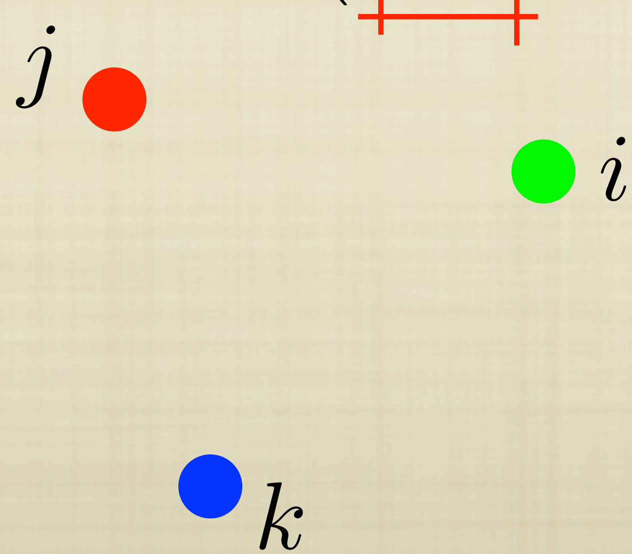

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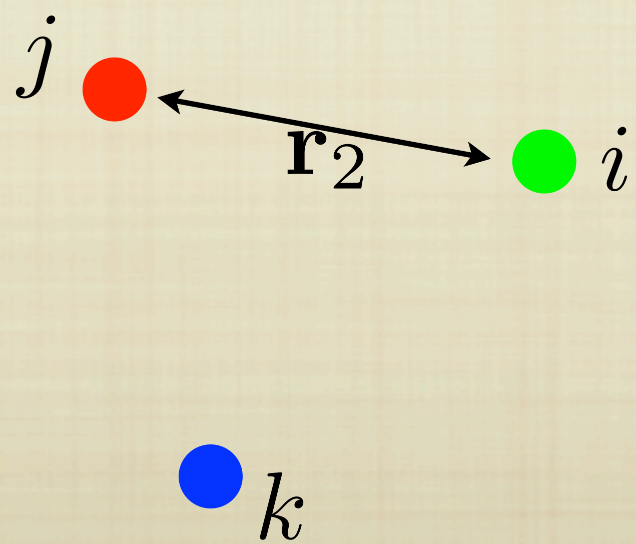

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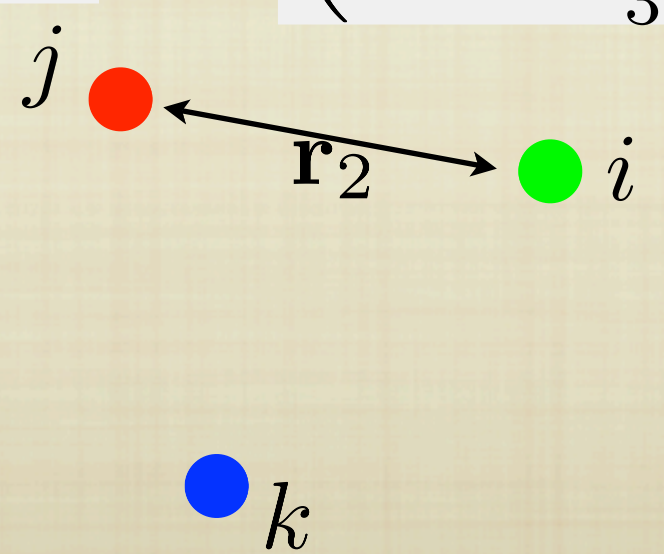
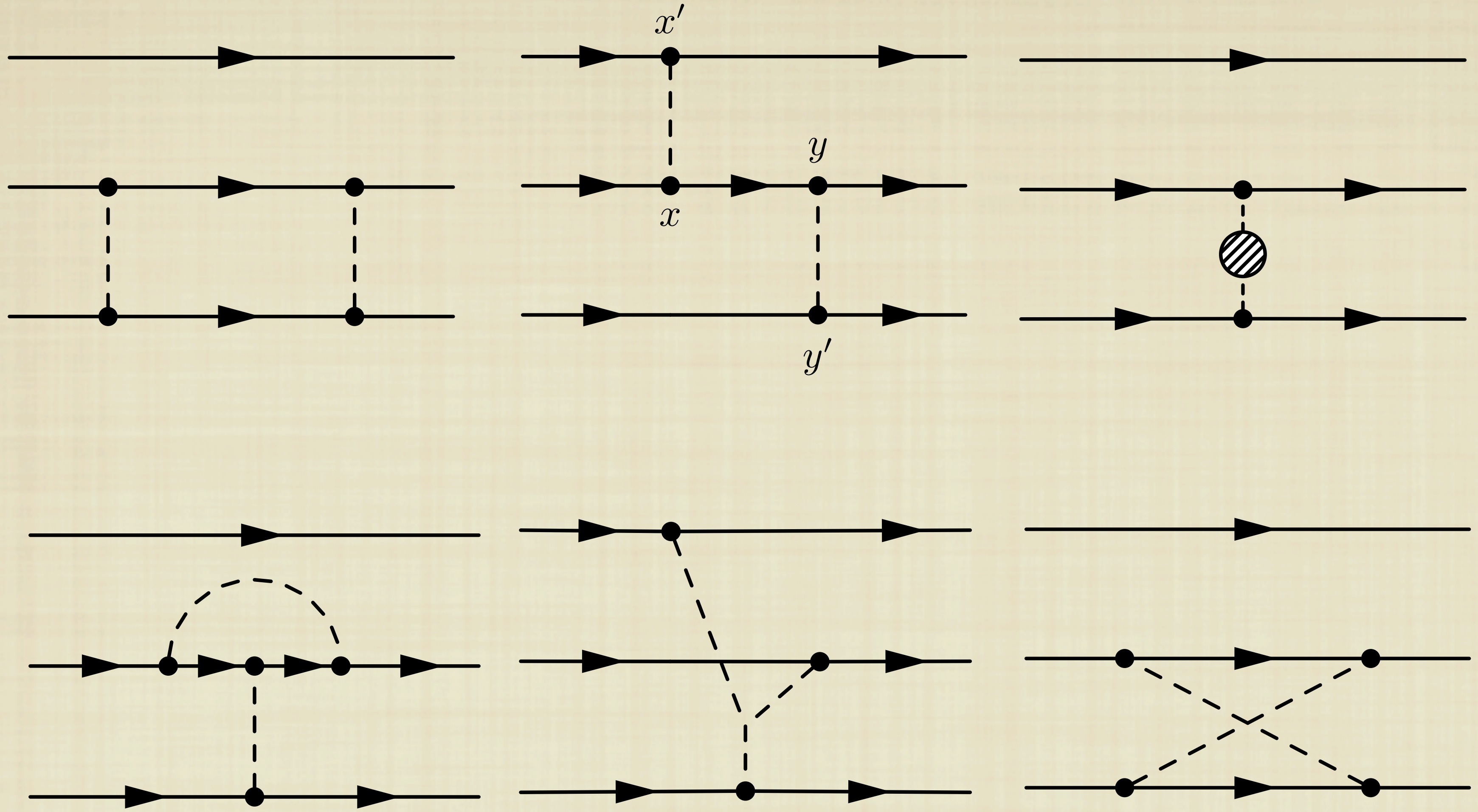


QQQ potential at NLO in Coulomb gauge



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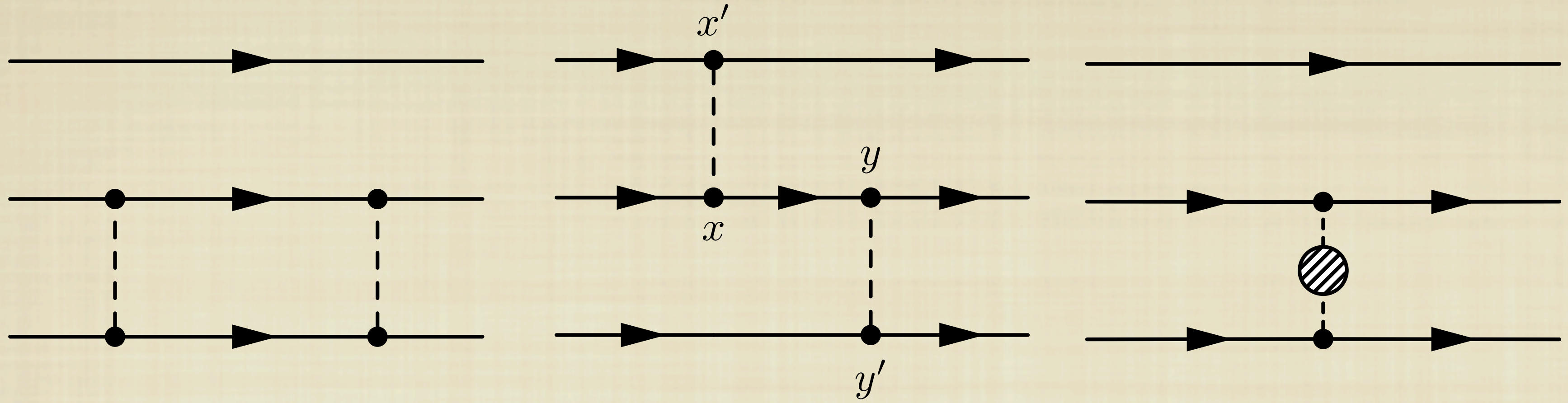

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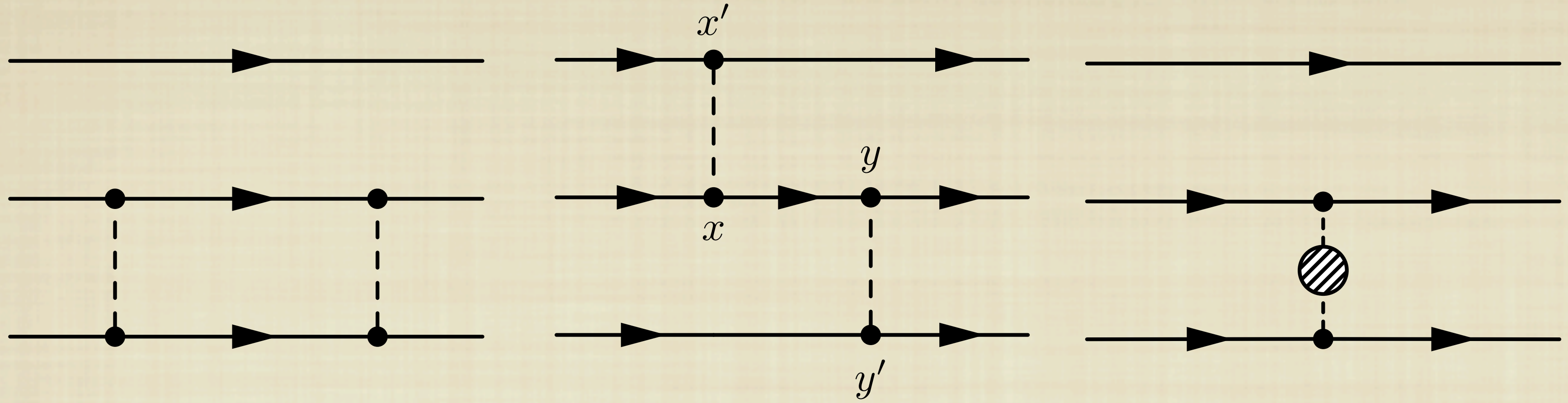

- Esponentiation

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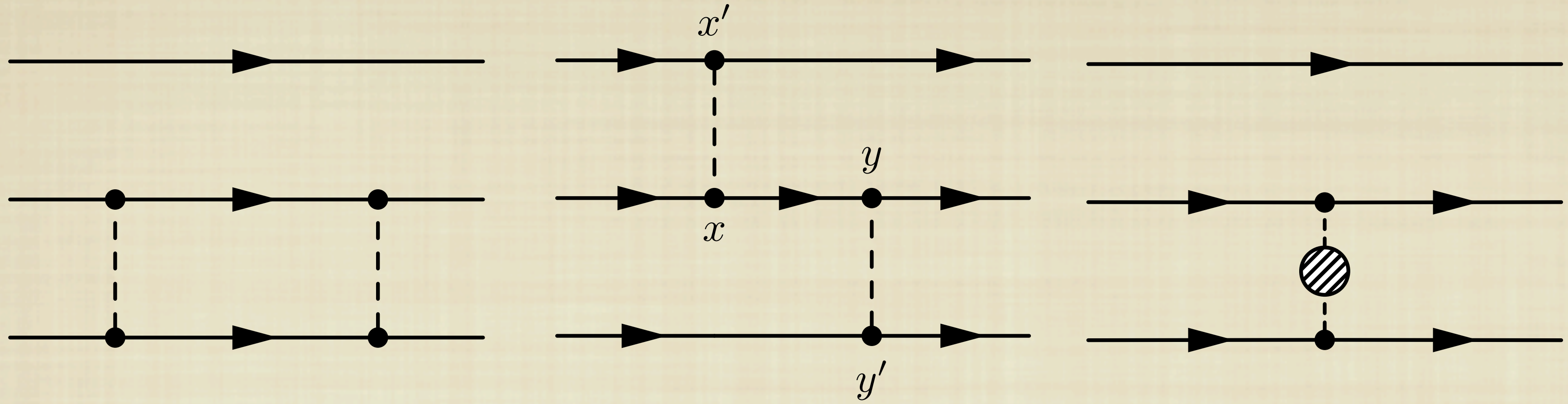

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$$V_{\mathcal{C}} = \lim_{T_W \to \infty} -\frac{1}{iT_W} \log \frac{\langle \mathcal{C}^u \, W \, \mathcal{C}^{v*} \rangle}{\langle S_{\mathcal{C}}^{uv} \rangle} \qquad e^{-iT_W V_{\mathcal{C}}} = 1 - iT_W V_{\mathcal{C}} - \frac{T_W^2}{2!} V_{\mathcal{C}}^2 + \dots$$

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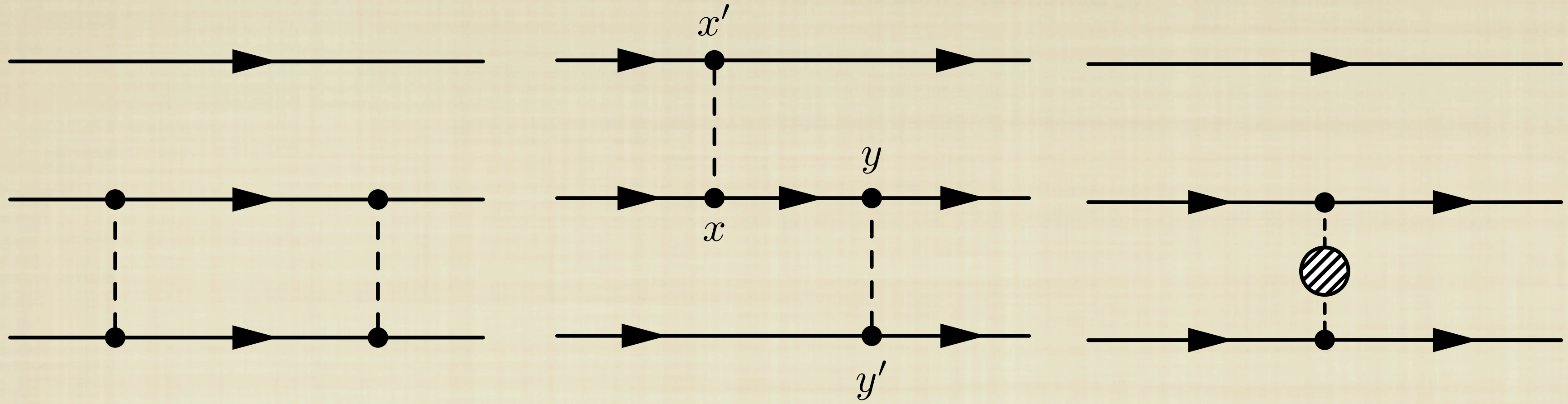

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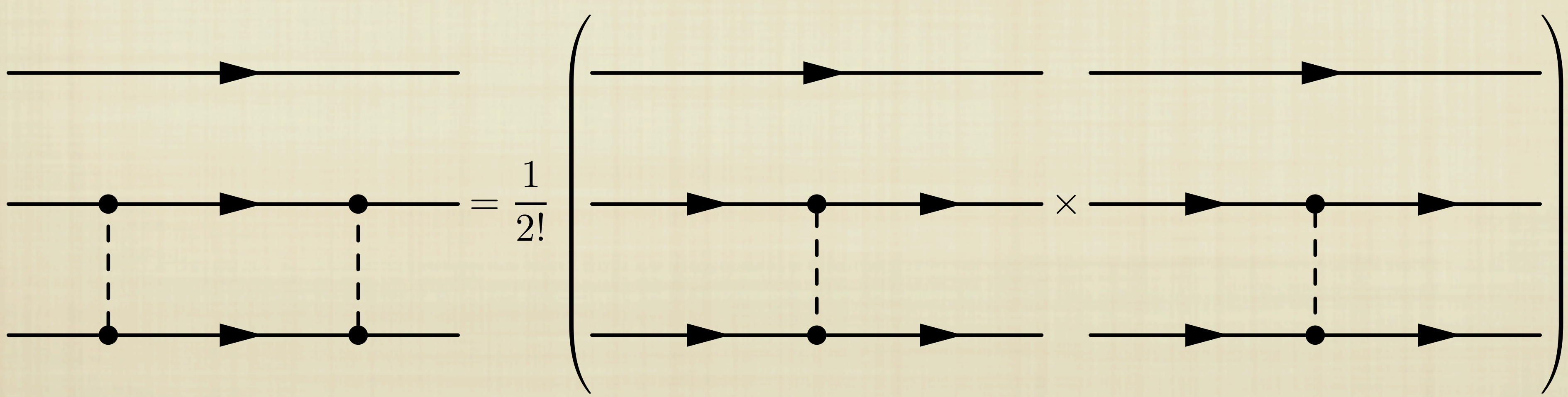

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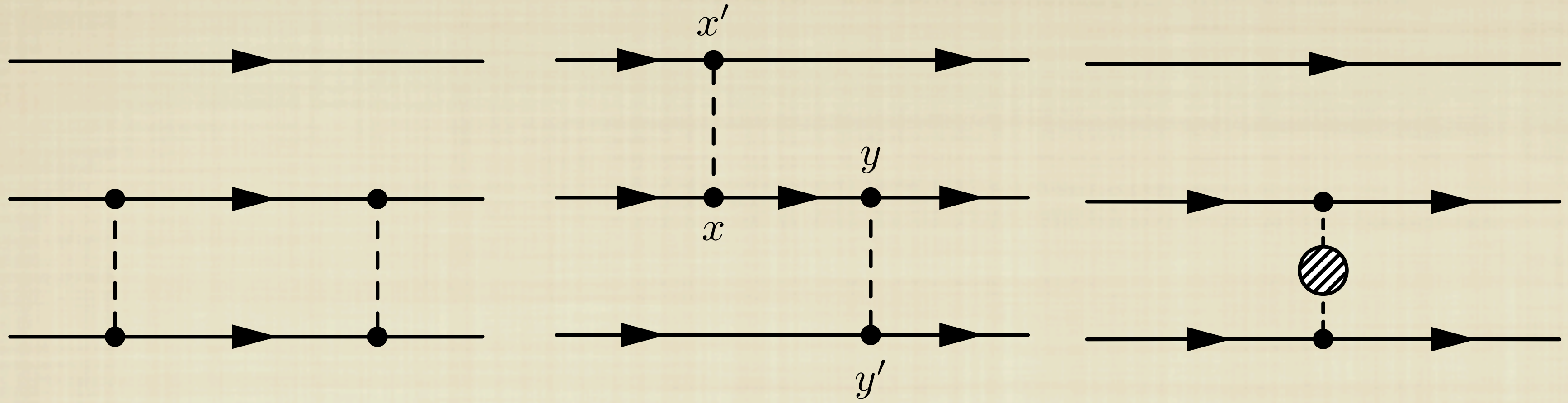

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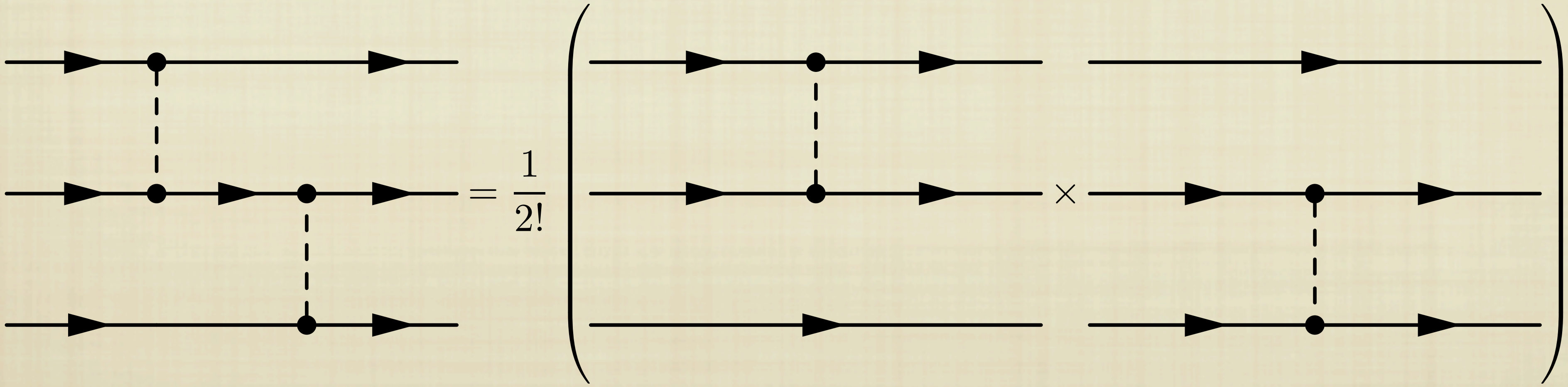

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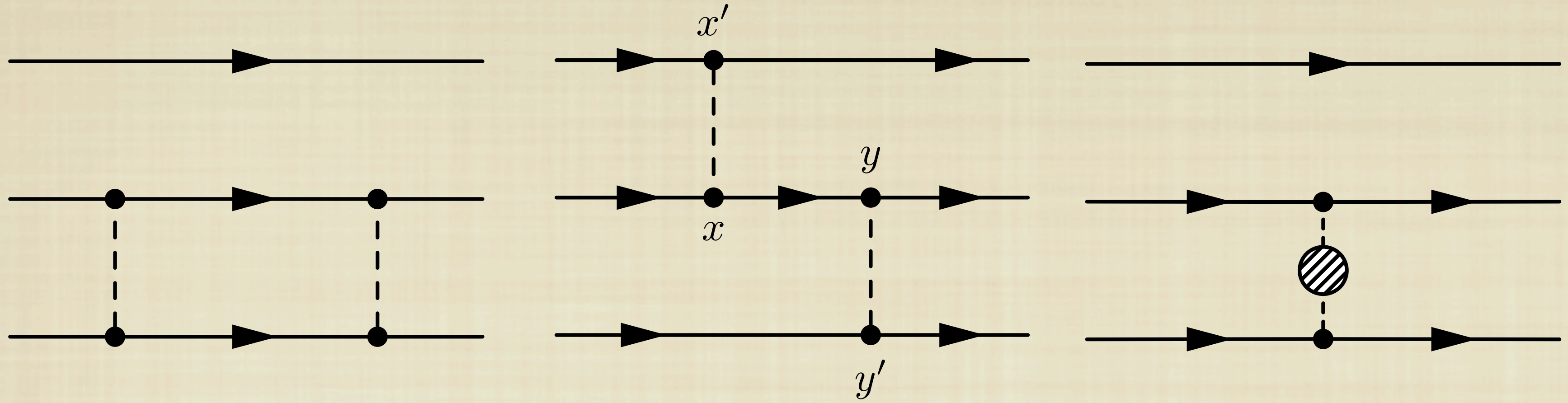

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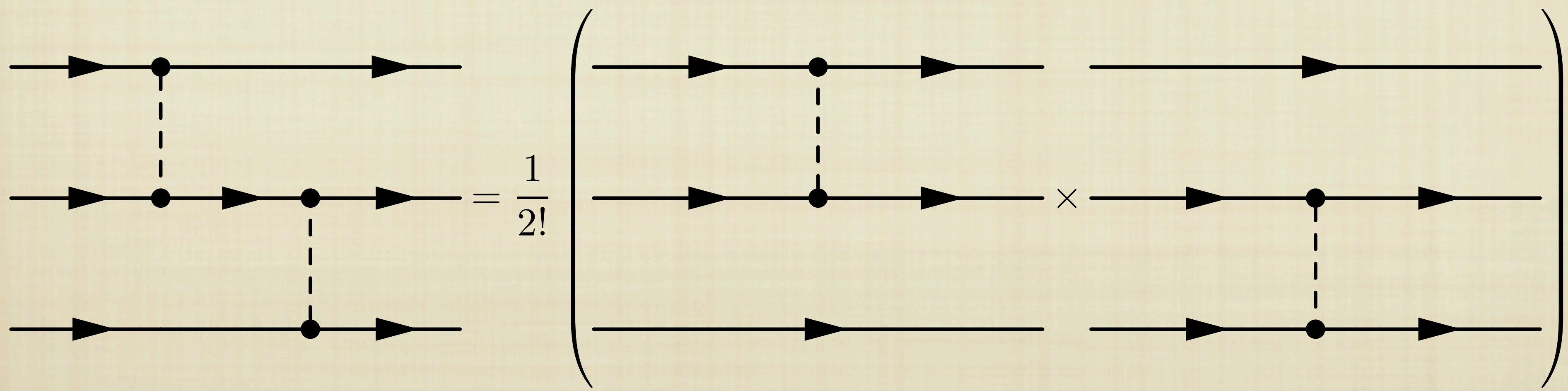
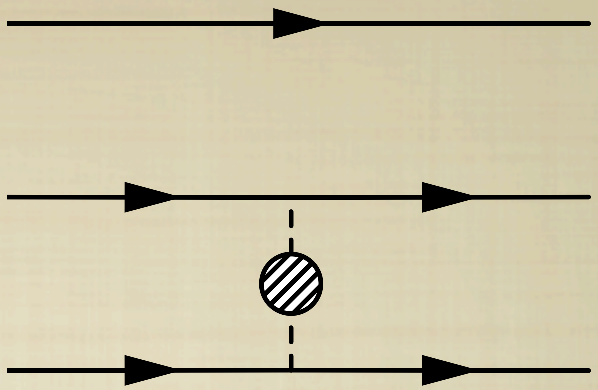
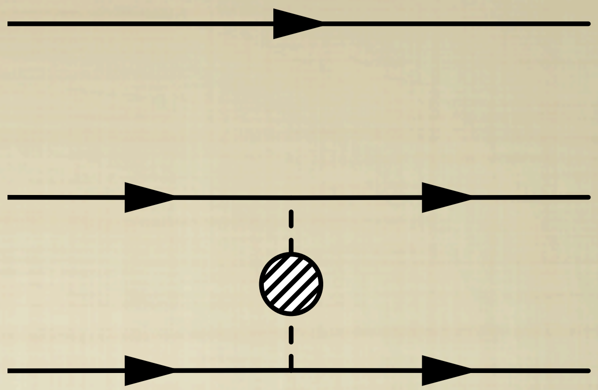


- The potential is still two body

QQQ potential at NLO





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$$V_{\mathcal{C}}^{1}(\mathfrak{r}) = \sum_{i=1}^{3} f_q^0(\mathcal{C}) \frac{\alpha_{\overline{MS}}(\mathbf{r}_q)}{|\mathbf{r}_q|} \left[1 + \frac{\alpha_{\overline{MS}}(\mathbf{r}_q)}{4\pi} \left(2\beta_0\gamma + a_1\right)\right]$$

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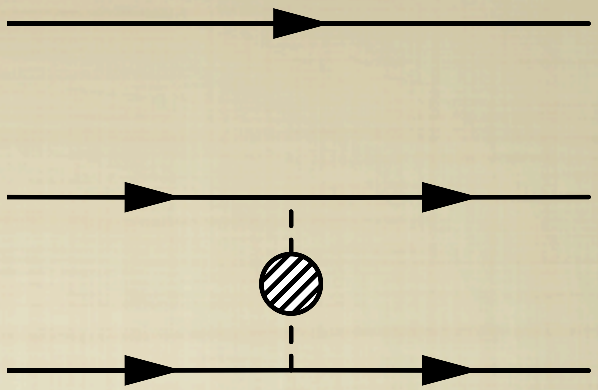

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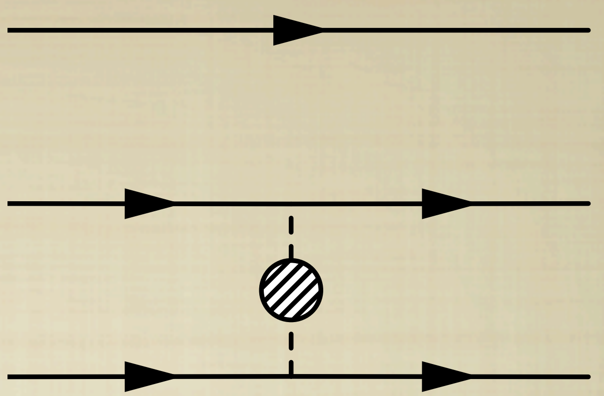

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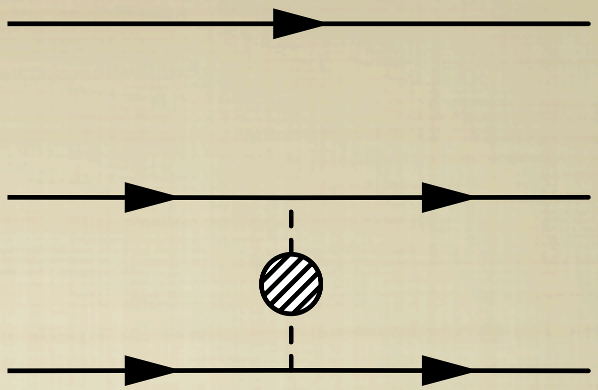

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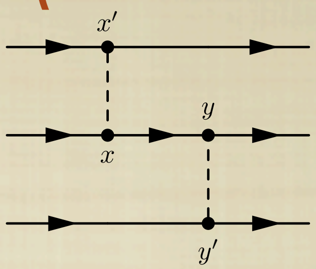
at NLO QQbar and QQQ potential only differ for the overall colour representation but the effective coupling of the potential is the same

$$\alpha_V(1/|\mathbf{r}_q|) = \alpha_s(1/|\mathbf{r}_q|) \left[1 + \frac{\alpha_s}{4\pi} (2\beta_0\gamma_E + a_1)\right],$$

At which order a genuine three body interaction (not generated by two body exponentiation like 

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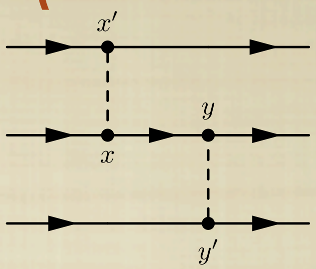

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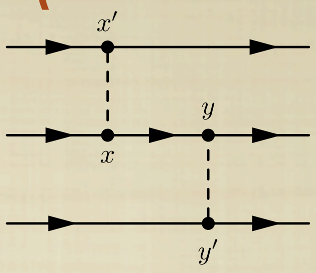

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WE WRITE

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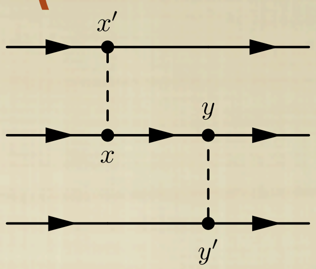

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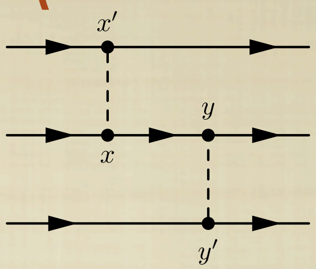

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$V_{\mathcal{C}}^{(2)}$ *is gauge invariant* $\to a_2^{\text{2body}}(\mathcal{C})$ *and* $V_{\mathcal{C}}^{\text{3body}}$ *are gauge invariant*

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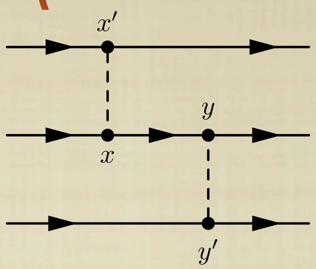

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$$V_{\mathcal{C}}^{(2)}(\mathfrak{r}) = V_{\mathcal{C}}^{3\mathrm{body}}(\mathfrak{r}) + \alpha_{\mathrm{s}}^3 \sum_{q=1}^{3} \frac{a_q^{2\mathrm{body}}(\mathcal{C})}{|\mathbf{r}_q|},$$

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- V^3body comes from diagrams with gluons attached to all 3 quark lines
- Many classes of diagrams
- The Coulomb gauge is again very useful

“Abelian”, exponentiating diagrams

$$e^{-iT_W V} = 1 - iT_W V - \frac{T_W^2}{2!}V^2 + i\frac{T_W^3}{3!}V^3 \dots$$

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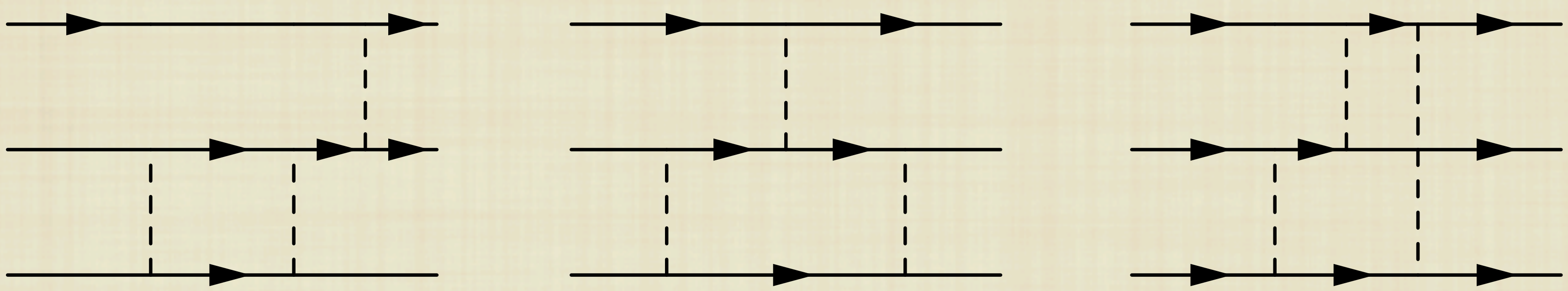

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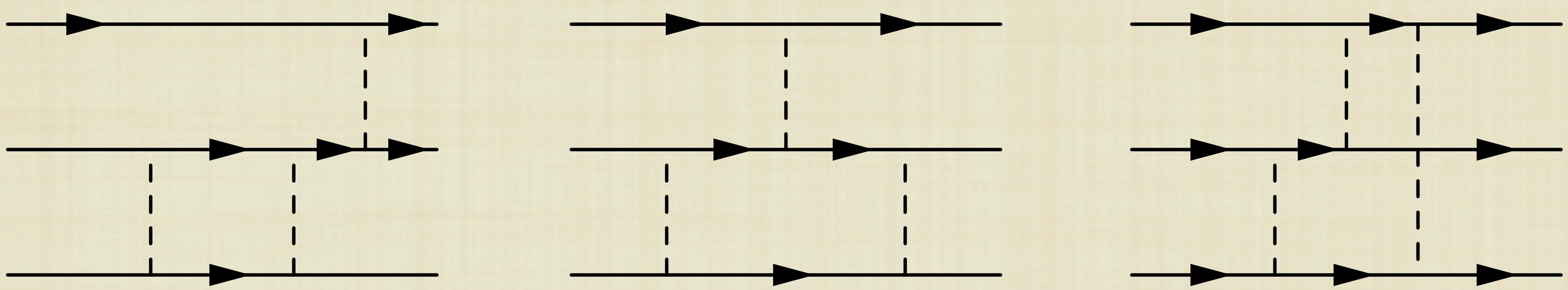
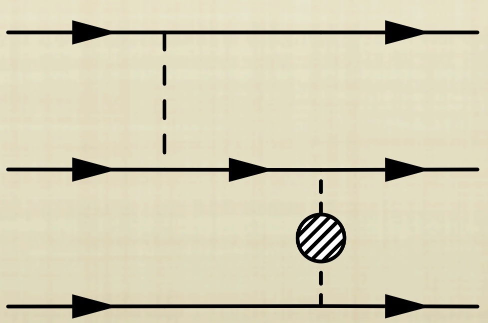
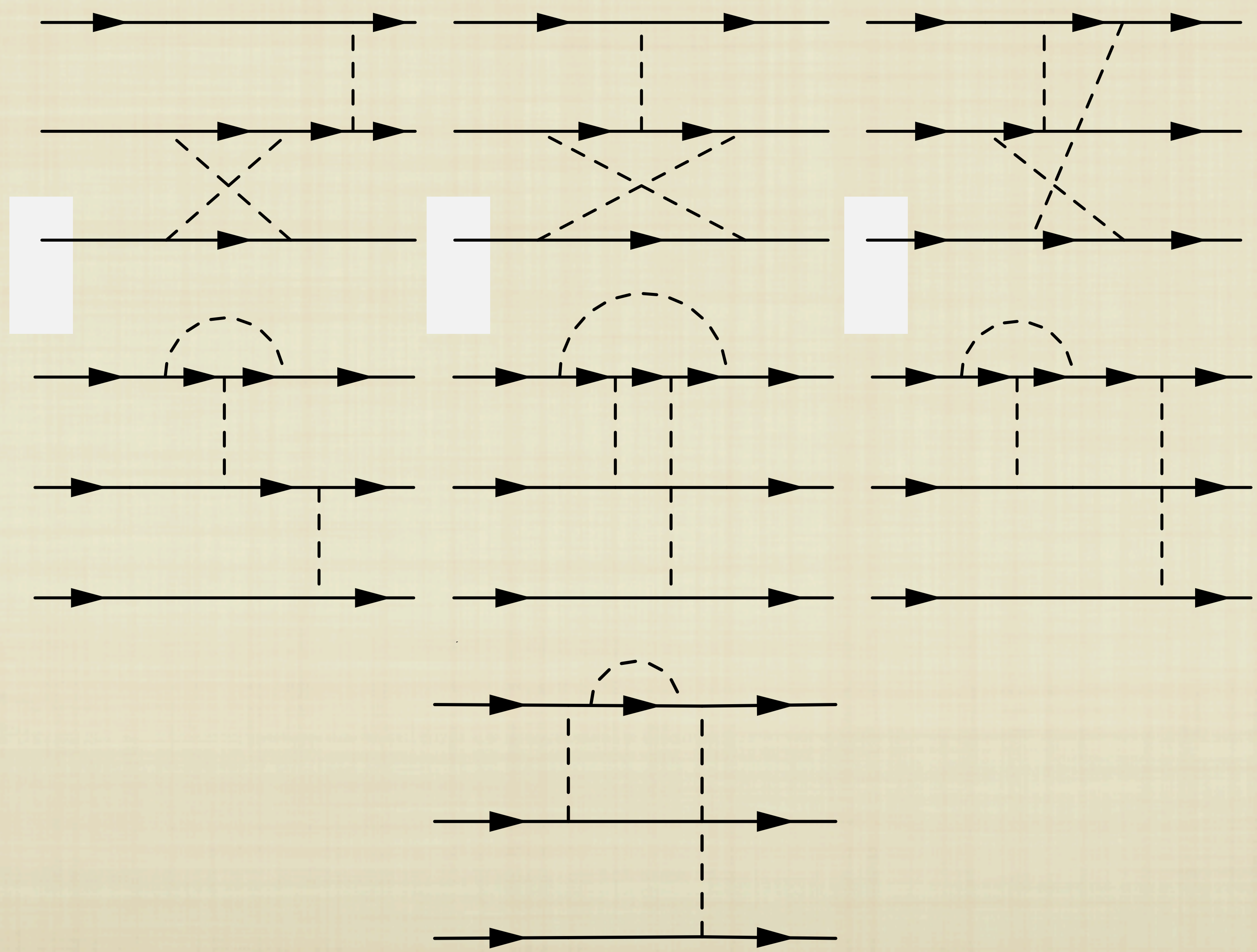


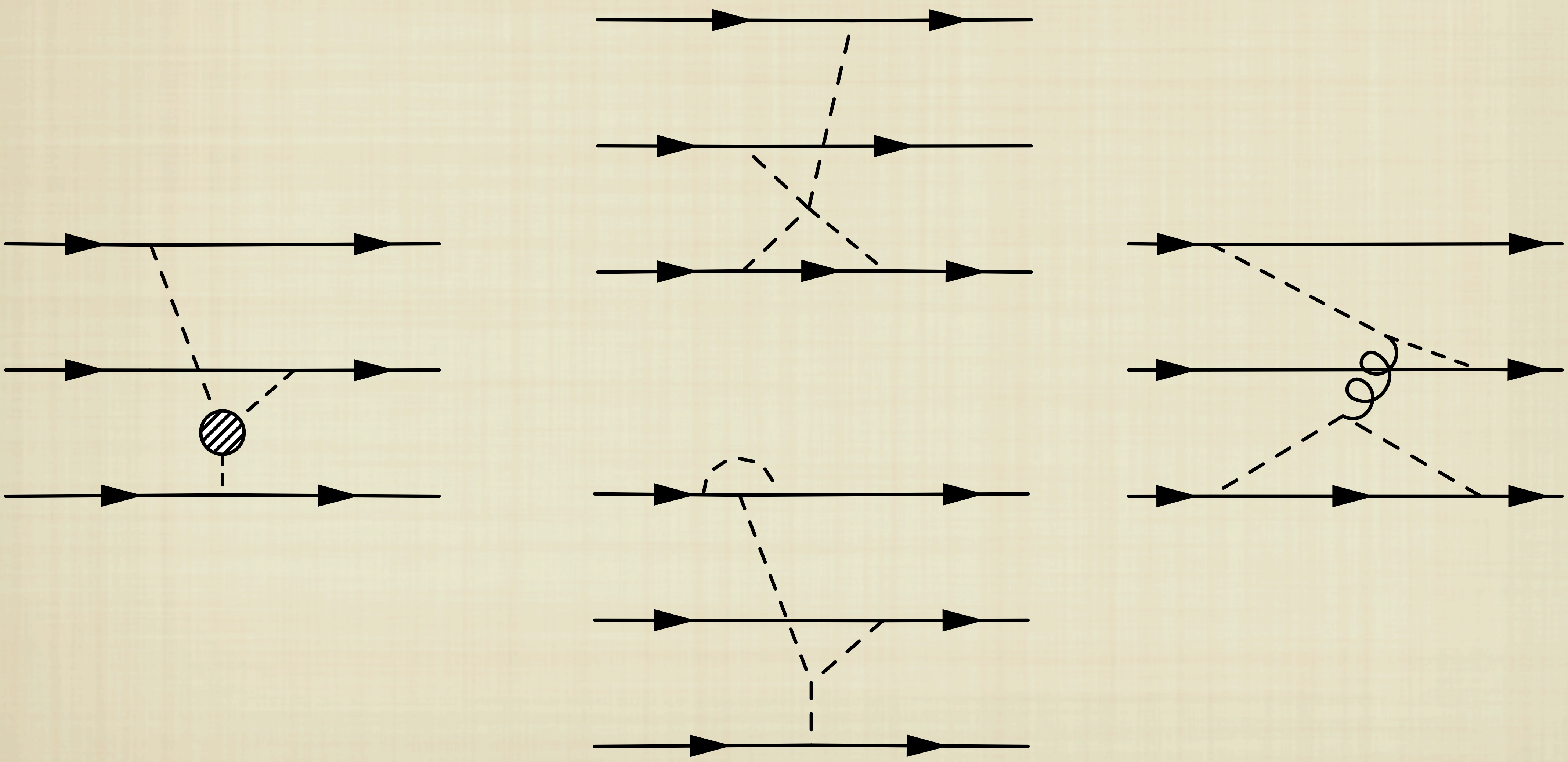
- Square terms of the one-loop potential



“Abelian” zero diagrams



Non-abelian zero diagrams



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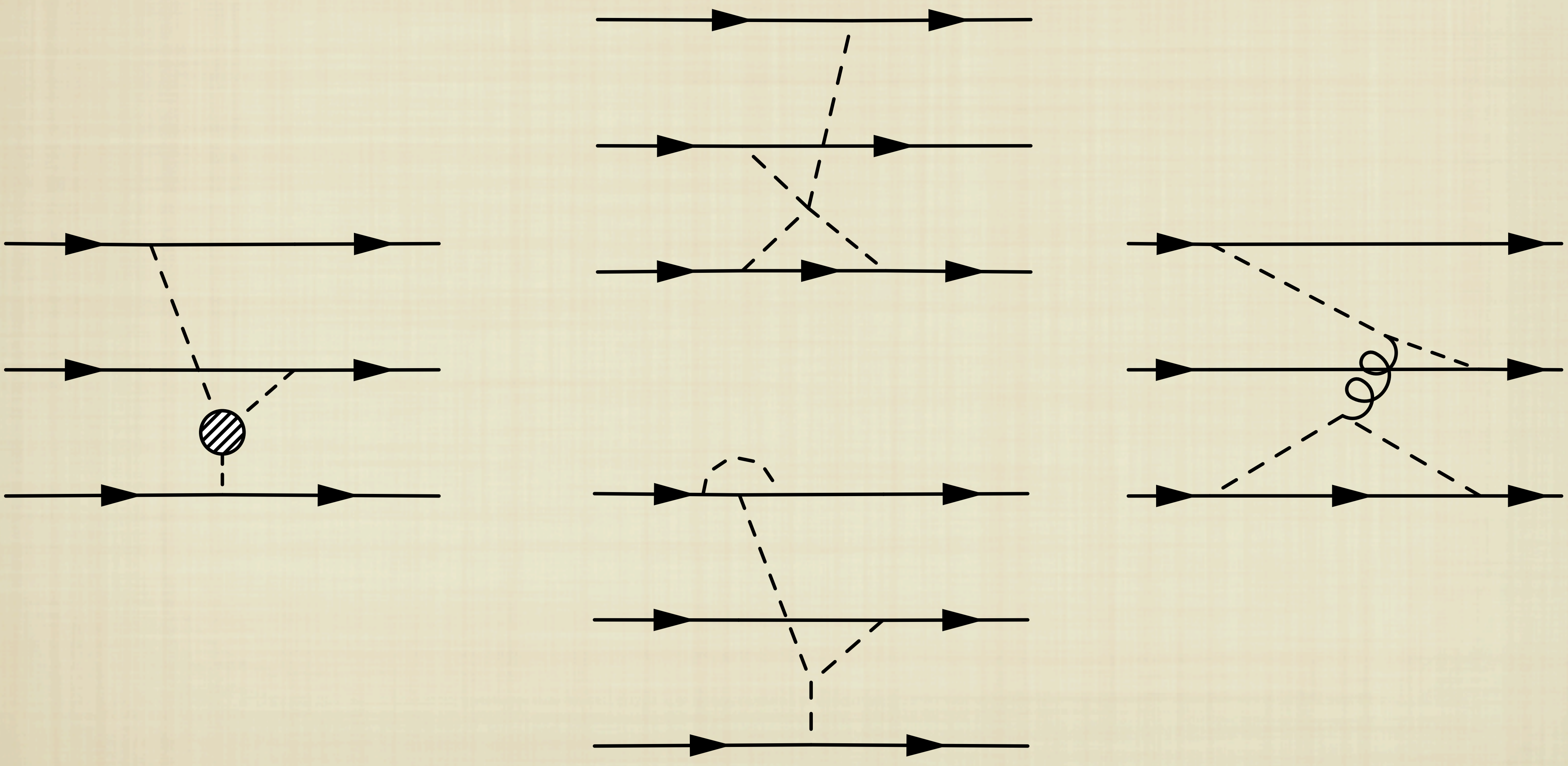

- What is left?

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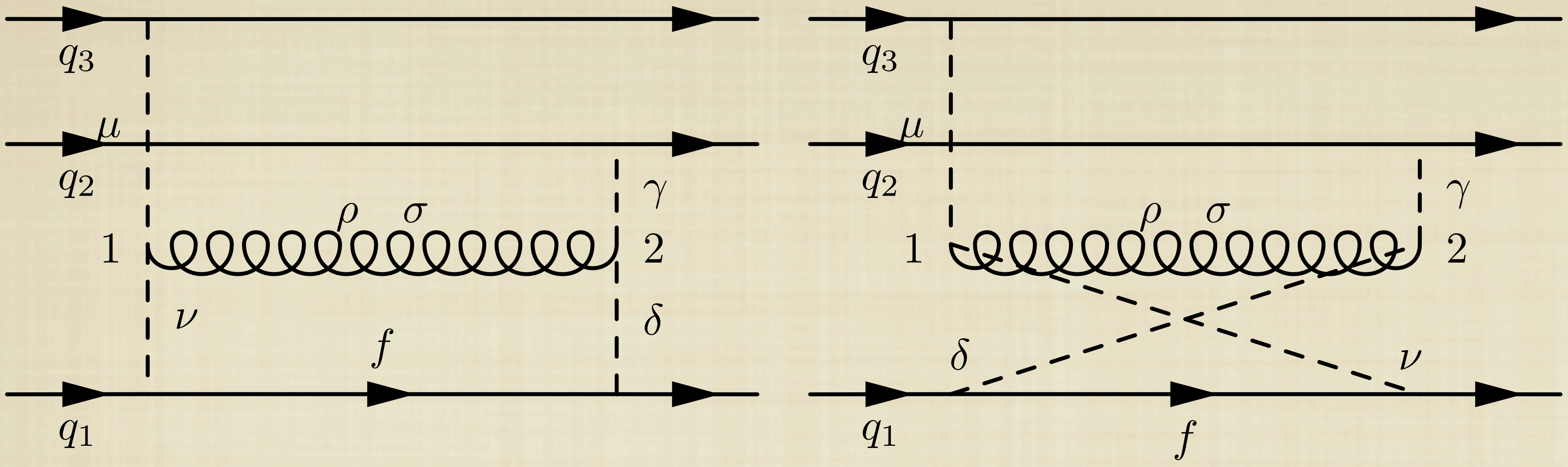

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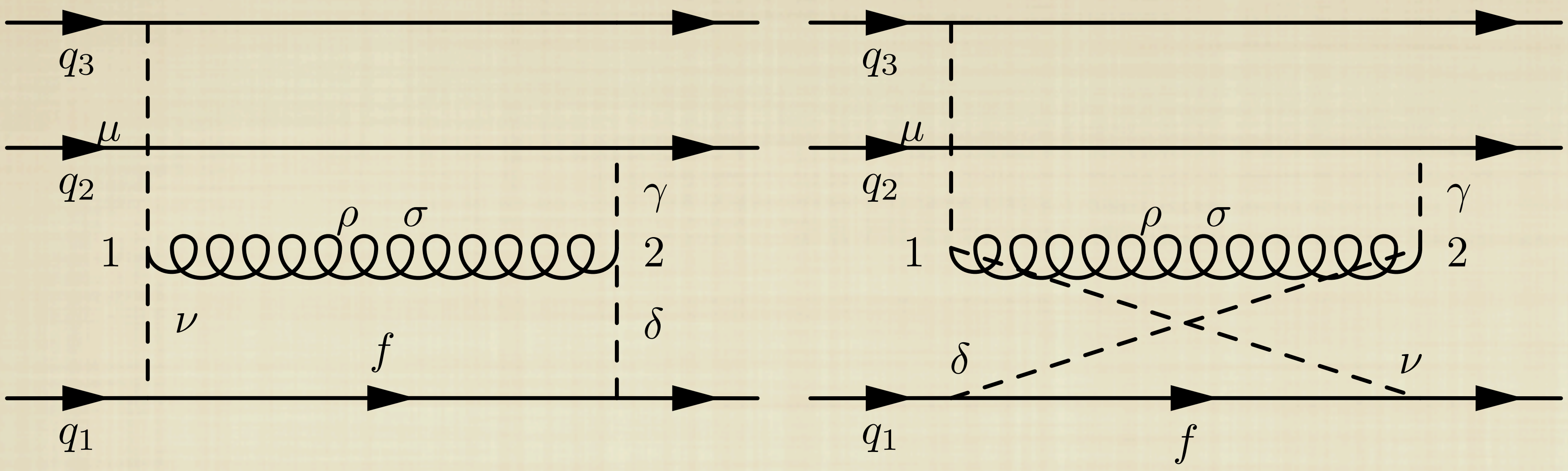

$$f_{\mathcal{H}}(S) = -\frac{1}{2} \quad \text{and} \quad f_{\mathcal{H}}(\Delta) = -\frac{1}{4}.$$

color factors equal for all 12 diagrams

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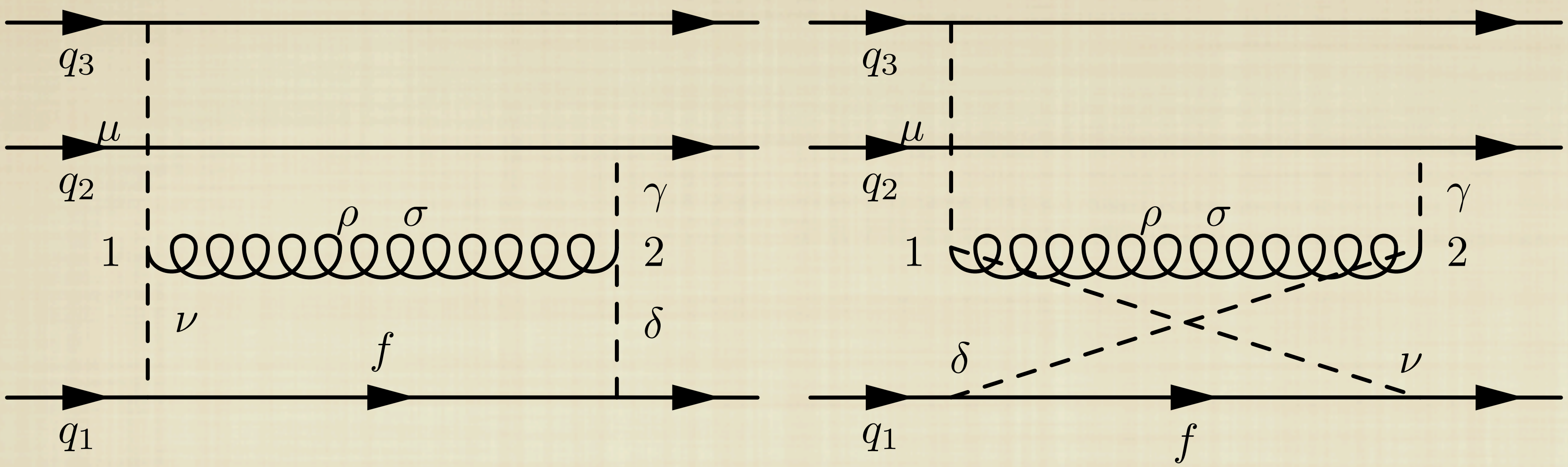

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complicate amplitude

$$\begin{aligned}\mathcal{H}_{\mathcal{C}}(\mathbf{q}_2,\mathbf{q}_3) &= -\frac{i f_{\mathcal{H}}(\mathcal{C}) g^6}{\mathbf{q}_2^2\mathbf{q}_3^2}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{4(\mathbf{q}_2\cdot\hat{\mathbf{k}}\,\mathbf{q}_3\cdot\hat{\mathbf{k}} - \mathbf{q}_2\cdot\mathbf{q}_3)}{\mathbf{k}^2(\mathbf{k}-\mathbf{q}_2)^2(\mathbf{k}+\mathbf{q}_3)^2} \\ &= \frac{i f_{\mathcal{H}}(\mathcal{C}) g^6}{8\mathbf{q}_2^2\mathbf{q}_3^2}\left[\frac{|\mathbf{q}_2+\mathbf{q}_3|}{|\mathbf{q}_2||\mathbf{q}_3|} + \frac{\mathbf{q}_2\cdot\mathbf{q}_3 + |\mathbf{q}_2||\mathbf{q}_3|}{|\mathbf{q}_2||\mathbf{q}_3||\mathbf{q}_2+\mathbf{q}_3|} - \frac{1}{|\mathbf{q}_2|} - \frac{1}{|\mathbf{q}_3|}\right].\end{aligned}$$

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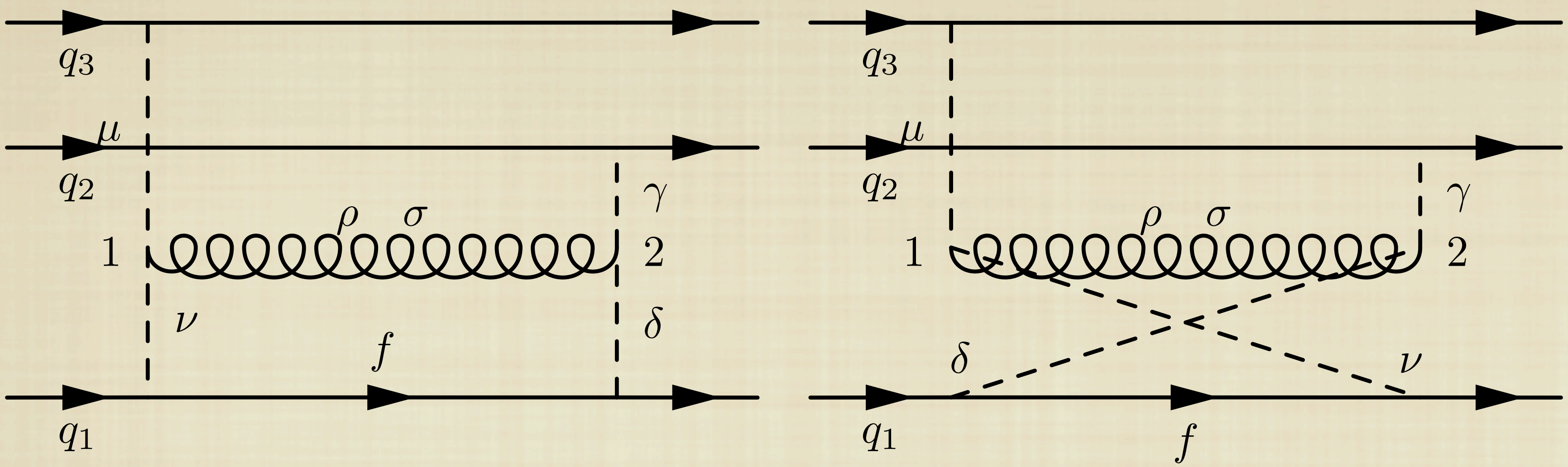

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complicate amplitude

$$\begin{aligned}\mathcal{H}_{\mathcal{C}}(\mathbf{q}_2, \mathbf{q}_3) &= -\frac{i f_{\mathcal{H}}(\mathcal{C}) g^6}{\mathbf{q}_2^2 \mathbf{q}_3^2} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{4(\mathbf{q}_2 \cdot \hat{\mathbf{k}}\, \mathbf{q}_3 \cdot \hat{\mathbf{k}} - \mathbf{q}_2 \cdot \mathbf{q}_3)}{\mathbf{k}^2 (\mathbf{k} - \mathbf{q}_2)^2 (\mathbf{k} + \mathbf{q}_3)^2} \\ &= \frac{i f_{\mathcal{H}}(\mathcal{C}) g^6}{8 \mathbf{q}_2^2 \mathbf{q}_3^2} \left[\frac{|\mathbf{q}_2 + \mathbf{q}_3|}{|\mathbf{q}_2||\mathbf{q}_3|} + \frac{\mathbf{q}_2 \cdot \mathbf{q}_3 + |\mathbf{q}_2||\mathbf{q}_3|}{|\mathbf{q}_2||\mathbf{q}_3||\mathbf{q}_2 + \mathbf{q}_3|} - \frac{1}{|\mathbf{q}_2|} - \frac{1}{|\mathbf{q}_3|} \right].\end{aligned}$$

the 3body potential in configuration space can be calculated numerically

Let us consider some simple geometries

Isosceles geometry in a plane $\quad |\mathbf{r}_2| = |\mathbf{r}_3| = r$ and $\hat{\mathbf{r}}_2 \cdot \hat{\mathbf{r}}_3 = \cos\theta.$

$$V^{\mathrm{tot}}_{\mathcal{H}\,\mathcal{C}}(r,\theta) = f_{\mathcal{H}}(\mathcal{C})\alpha_{\mathrm{s}}^3 \frac{c_{\mathcal{H}}(\theta)}{r}.$$

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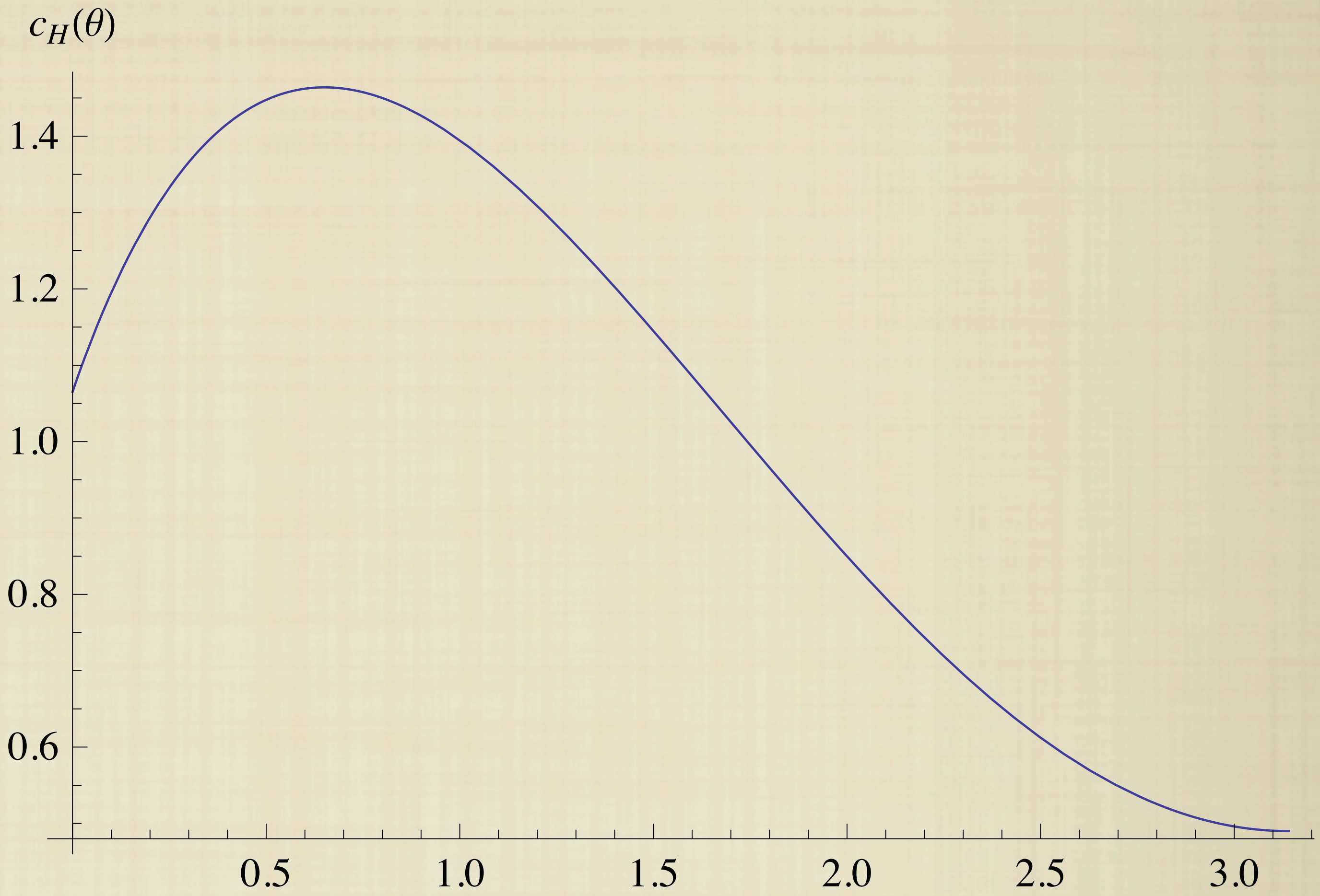

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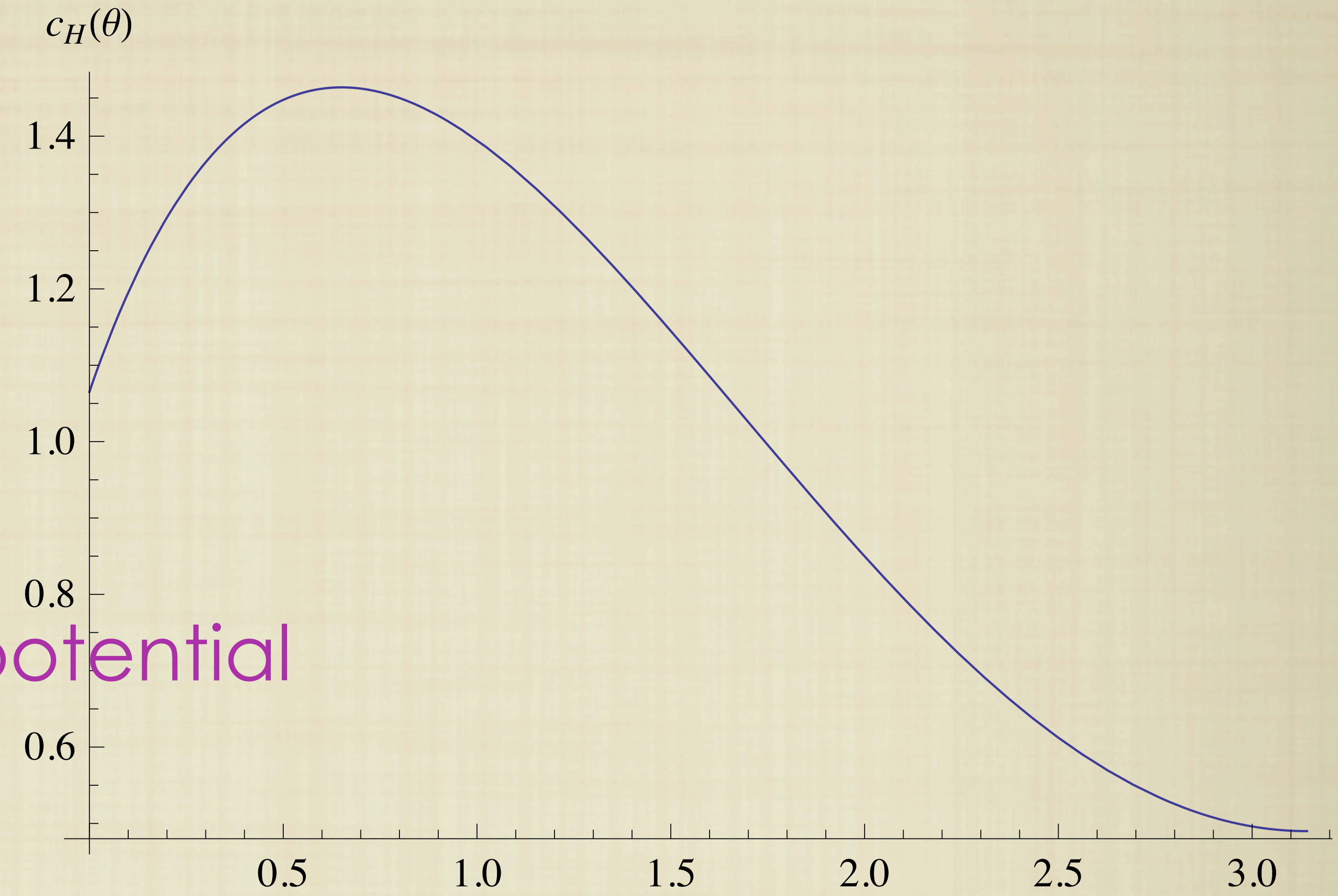

attractive contribution to the potential

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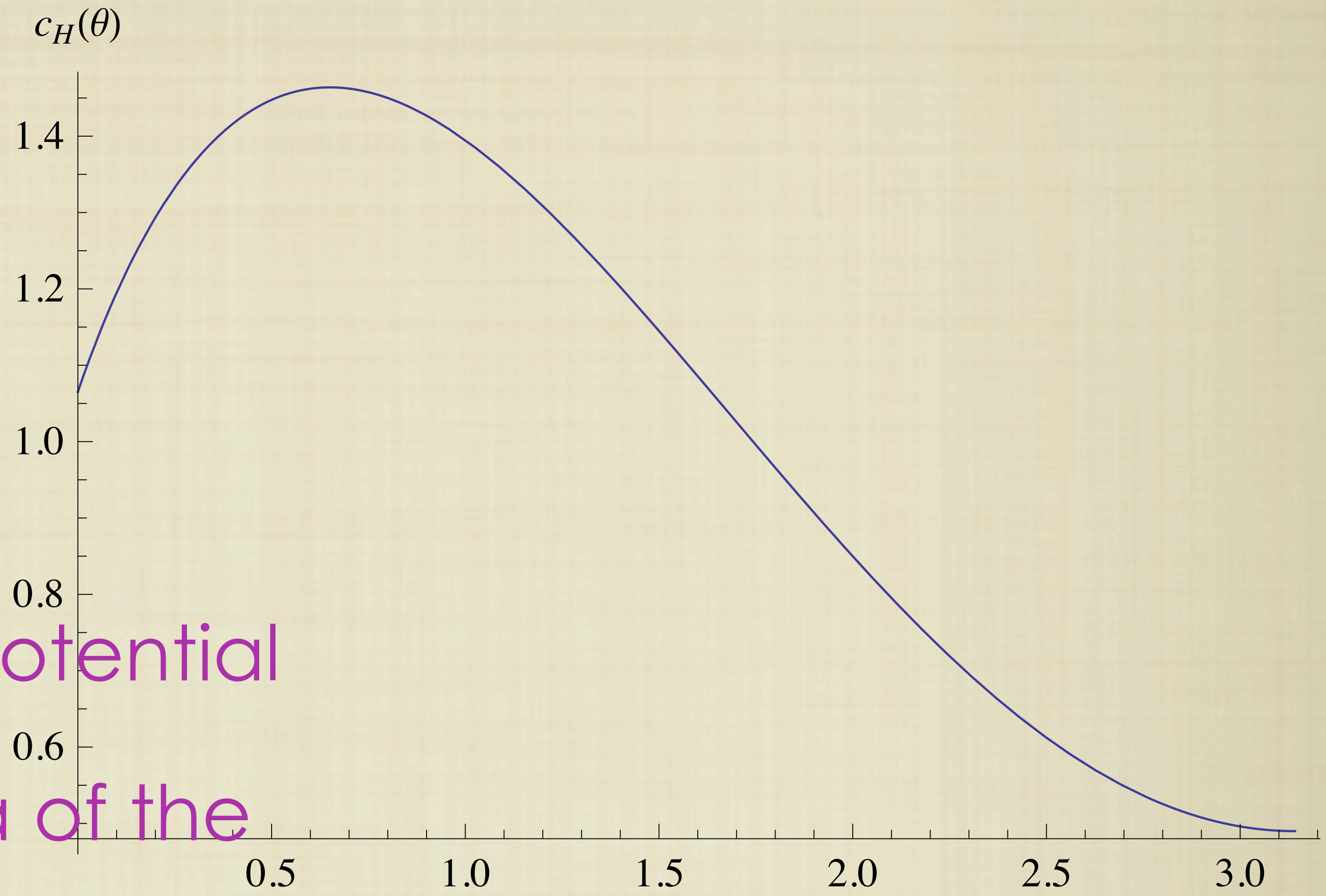

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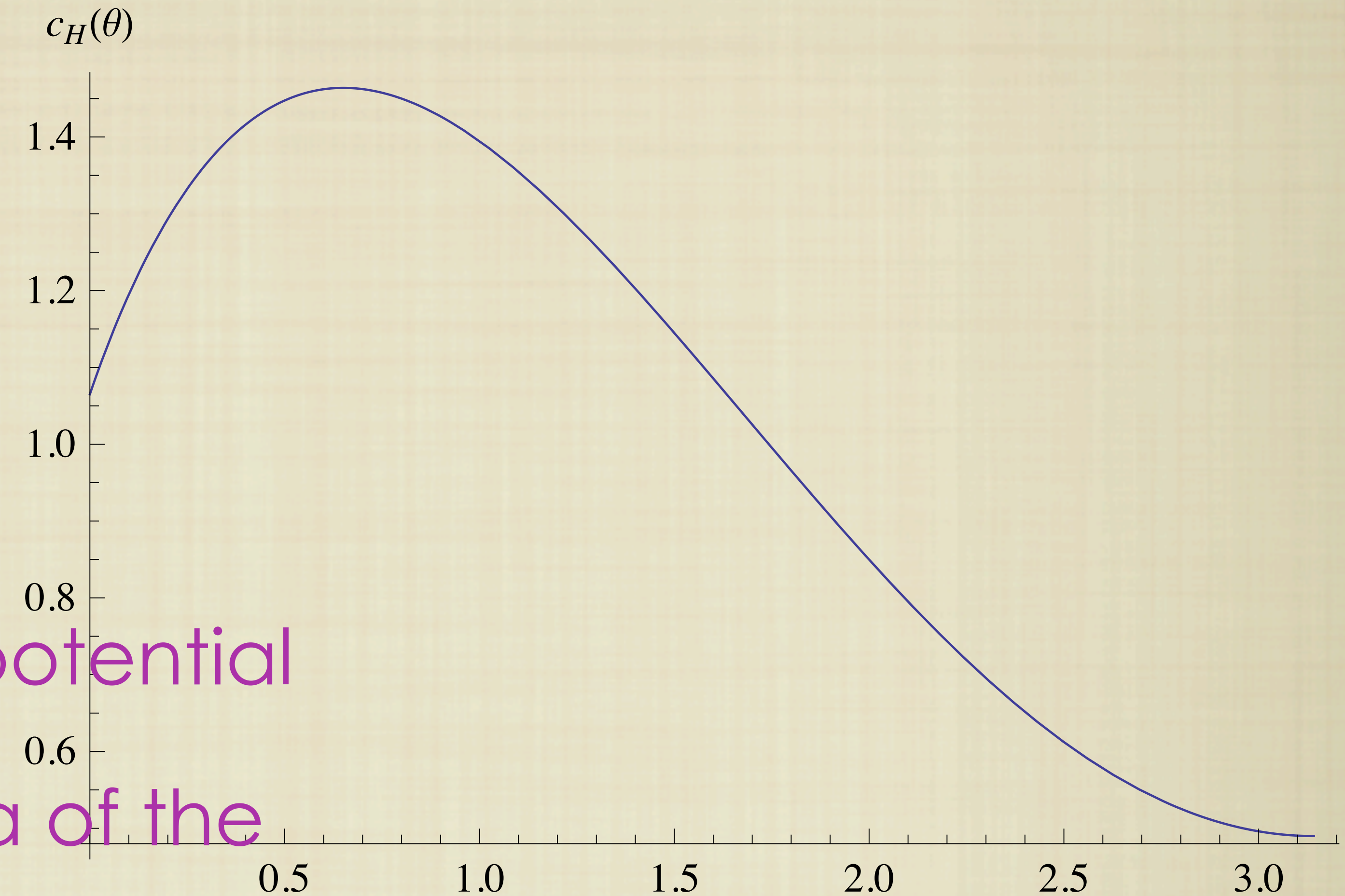

attractive contribution to the potential

weak dependence on theta of the 3body potential

may indicate the onset of a smooth transition towards the long distance Y shaped three body potential seen in the lattice data?

Let us consider some simple geometries

$\theta = \pi/3$: *planar equilateral geometry*

In the equilateral case, we have $c_{\mathcal{H}}(\pi/3) \approx 1.377$.

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We can compare the relative magnitude of the three-body contribution to the tree level potential. For the singlet

$$\frac{V^{\rm tot}_{\mathcal{H}\,s}(r)}{V_s^{(0)}(r)} = \frac{c_{\mathcal{H}}(\pi/3)}{4}\alpha_{\rm s}^2(1/r) \approx \frac{\alpha_{\rm s}^2(1/r)}{2.90}.$$

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using $\alpha_{\rm s}$ at one loop, $V^{\rm tot}_{\mathcal{H}\,s}(r)$ may become as large as one sixth of the tree-level Coulomb potential in the region around 0.3 fm, where, at least in the $Q\overline{Q}$ case, perturbation theory still holds

Let us consider some simple geometries

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$Generic\ geometry$

In the most general geometry the three body potential depends on two coordinates, we may choose one of them to be L_min, leaving the other not specified

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(B.1) Planar lattice geometry with two fixed quarks

In Fig 10, we plot the three-body potential obtained by placing the three quarks in a plane (x, y), fixing the position of the first quark in $(0, 0)$, the second one in $(1, 0)$ and moving the third one in the lattice $(0.5+0.125\, n_x, 0.125\, n_y)$ with $n_x \in \{0, 1, ..., 20\}$ and $n_y \in \{0, 1, ..., 24\}$. The plot clearly shows the dependence on the geometry at fixed L, however, the dependence is weaker than in the two-body case.

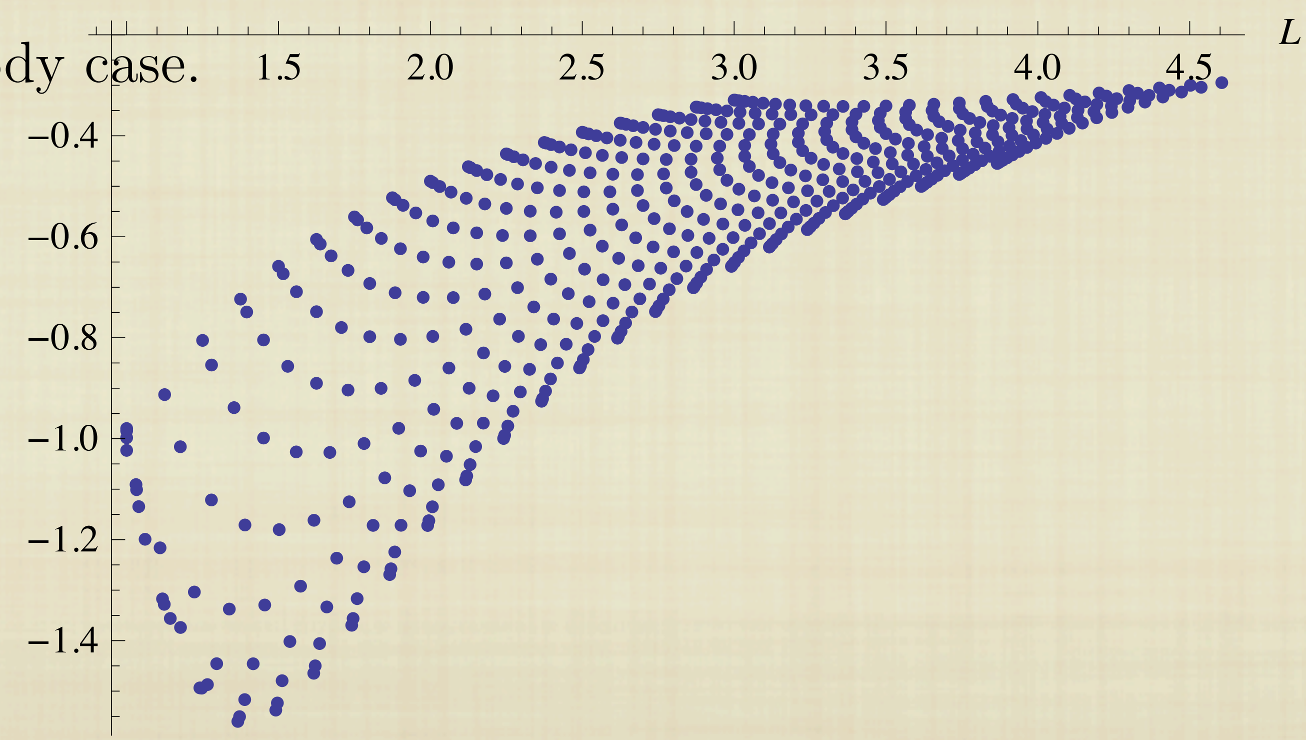

FIG. 10: The normalized three-body potential, $V^{\rm tot}_{\mathcal{HC}}(L, ...)/(-f_{\mathcal{H}}(\mathcal{C})\alpha_s^3)$, plotted as function of L

Let us consider some simple geometries

Generic geometry

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Three-dimensional lattice geometry with the three quarks moving along the axes

[28] T. T. Takahashi and H. Suganuma, Phys. Rev. **D70**, 074506 (2004), hep-lat/0409105.

In the lattice calculation of Ref. [28], the three quarks were located along the axes of a three-dimensional lattice, namely at $(n_x, 0, 0)$, $(0, n_y, 0)$ and $(0, 0, n_z)$, with $n_x \in \{0, 1, ..., 6\}$ and $n_y, n_z \in \{1, ..., 6\}$. For the sake of comparison, we consider the same geometry and plot the corresponding three-body potential in Fig. 11. The plot shows a weak dependence on the geometry: much weaker than in the two-body case, but also somewhat weaker than in the geometry considered in (B.1).

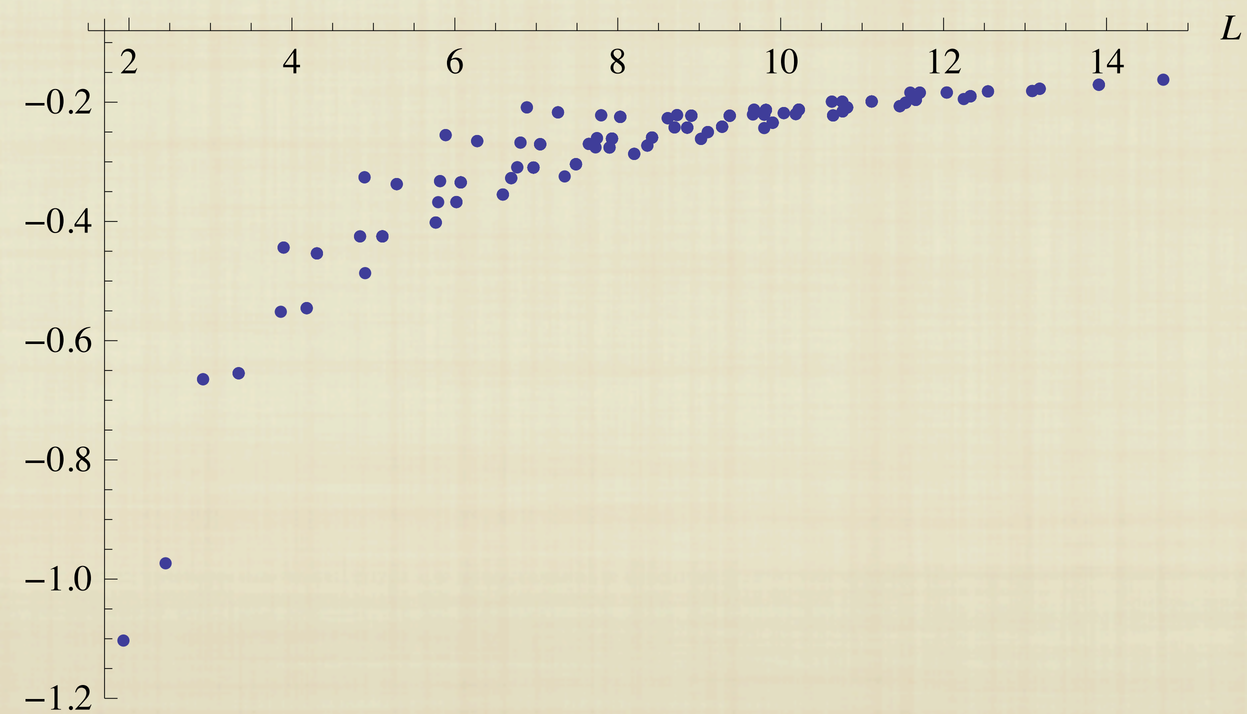

FIG. 11: The normalized three-body potential, $V^{\rm tot}_{\mathcal{HC}}(L, ...)/(-f_{\mathcal{H}}(\mathcal{C})\alpha_s^3)$, plotted as function of L

Full singlet (2 and 3 body) QQQ potential at N^2LO

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in the singlet case

$$V_s^{(2)}(\mathfrak{r}) = V_s^{3\text{body}}(\mathfrak{r}) + \alpha_{\rm s}^3 a^{2\text{body}}(S) \sum_{q=1}^{3} \frac{1}{|\mathbf{r}_q|}.$$

$a^{2\text{body}}(S)$ is independent of the geometry of the three quarks:

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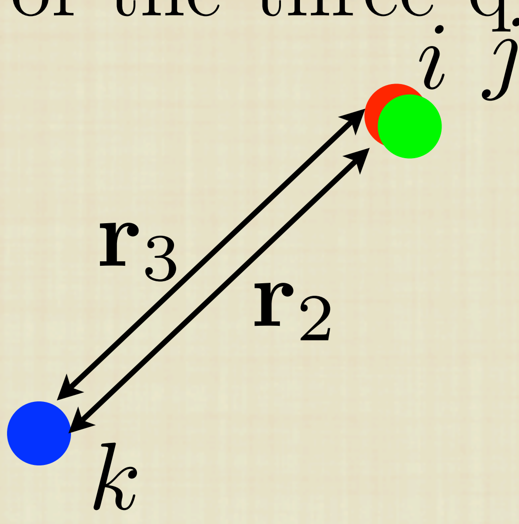

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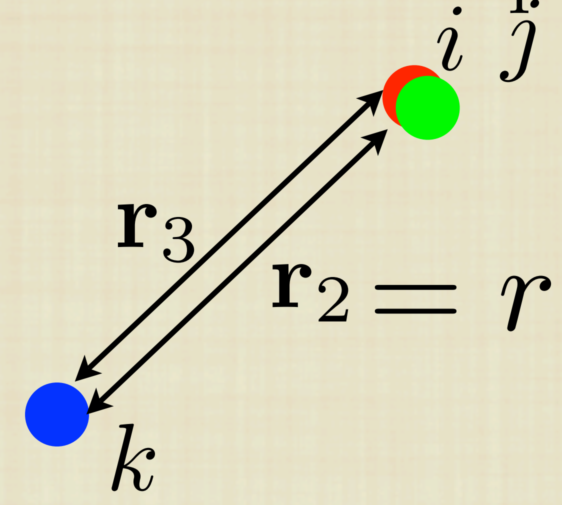

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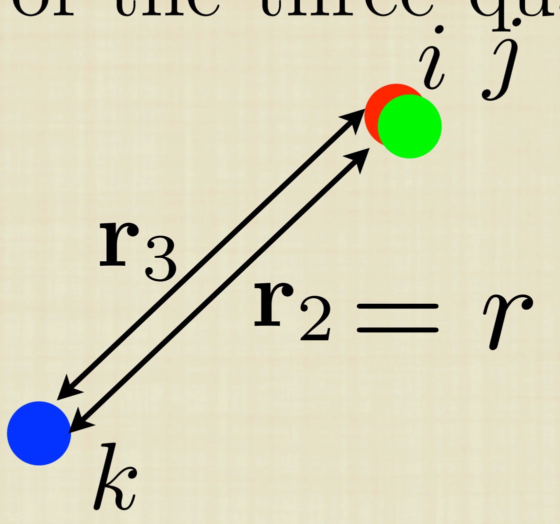

$$V_s^{(2)}(r) = -\left(3 - \frac{\pi^2}{4}\right)\frac{\alpha_s^3}{r} + 2\alpha_s^3 \frac{a^{2\text{body}}(S)}{r},$$

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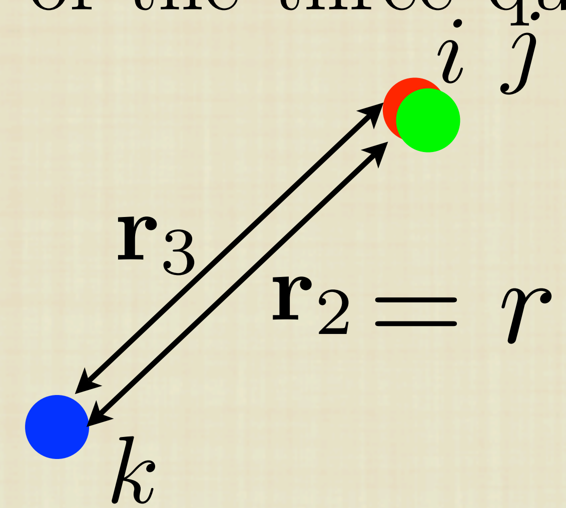

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Full singlet (2 and 3 body) QQQ potential at N^2LO

in the singlet case

$$V_s^{(2)}(\mathfrak{r}) = V_s^{\rm 3body}(\mathfrak{r}) + \alpha_{\rm s}^3 a^{\rm 2body}(S) \sum_{q=1}^{3} \frac{1}{|\mathbf{r}_q|}.$$

$a^{\rm 2body}(S)$ is independent of the geometry of the three quarks:

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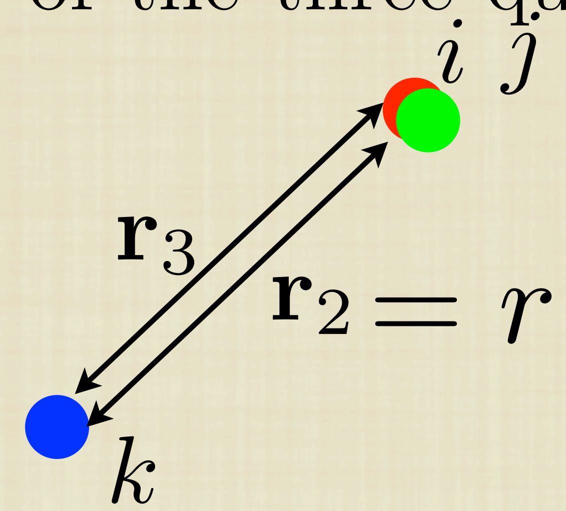

$$V_s^{(2)}(r) = -\left(3 - \frac{\pi^2}{4}\right)\frac{\alpha_{\rm s}^3}{r} + 2\alpha_{\rm s}^3 \frac{a^{\rm 2body}(S)}{r}, = V^{(2)}_{Q\bar{Q}}(r)$$

$$a^{\rm 2body}(S) = -\frac{2}{3}\frac{1}{(4\pi)^2}\left[a_2 - 36\pi^2 + 3\pi^4 + \left(\frac{\pi^2}{3} + 4\gamma_E^2\right)\beta_0^2 + \gamma_E\left(4a_1\beta_0 + 2\beta_1\right)\right].$$

Full QQQ Potential at N^2LO
two and three bodies parts

$$
\begin{aligned}
V_s(\mathbf{r}) \;=\; & -\frac{2}{3}\sum_{q=1}^{3}\frac{\alpha_{\rm s}(1/|\mathbf{r}_q|)}{|\mathbf{r}_q|}\left\{1+\frac{\alpha_{\rm s}(1/|\mathbf{r}_q|)}{4\pi}\left[\frac{31}{3}+22\gamma_E-\left(\frac{10}{9}+\frac{4}{3}\gamma_E\right)n_f\right]\right. \\
& +\left(\frac{\alpha_{\rm s}(1/|\mathbf{r}_q|)}{4\pi}\right)^2\left[+66\zeta(3)+484\gamma_E^2+\frac{1976}{3}\gamma_E+\frac{3}{4}\pi^4+\frac{121}{3}\pi^2+\frac{4343}{18}\right. \\
& -\left(\frac{52}{3}\zeta(3)+\frac{176}{3}\gamma_E^2+\frac{916}{9}\gamma_E+\frac{44}{9}\pi^2+\frac{1229}{27}\right)n_f \\
& \left.\left.+\left(\frac{16}{9}\gamma_E^2+\frac{80}{27}\gamma_E+\frac{4}{27}\pi^2+\frac{100}{81}\right)n_f^2\right]\right\} \\
& -\alpha_{\rm s}\left(\frac{\alpha_{\rm s}}{4\pi}\right)^2\left[v_{\mathcal{H}}(\mathbf{r}_2,\mathbf{r}_3)+v_{\mathcal{H}}(\mathbf{r}_1,-\mathbf{r}_3)+v_{\mathcal{H}}(-\mathbf{r}_2,-\mathbf{r}_1)\right]
\end{aligned}
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Full QQQ Potential at N^2LO two and three bodies parts

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where $v_{\mathcal{H}}(\mathbf{r}_2,\mathbf{r}_3) \;=\; 16\pi\hat{\mathbf{r}}_2\cdot\hat{\mathbf{r}}_3\int_0^1 dx\int_0^1 dy\,\frac{1}{R}\left[\left(1-\frac{M^2}{R^2}\right)\arctan\frac{R}{M}+\frac{M}{R}\right]\;+16\pi\hat{\mathbf{r}}_2{}^i\hat{\mathbf{r}}_3{}^j$
$\times\int_0^1 dx\int_0^1 dy\,\frac{\hat{\mathbf{R}}^i\hat{\mathbf{R}}^j}{R}\left[\left(1+3\frac{M^2}{R^2}\right)\arctan\frac{R}{M}-3\frac{M}{R}\right]$, with $\mathbf{R}=x\mathbf{r}_2-y\mathbf{r}_3$, $R=|\mathbf{R}|$ and

$$M \;=\; |\mathbf{r}_2|\sqrt{x(1-x)}+|\mathbf{r}_3|\sqrt{y(1-y)}.$$

Full QQ antitriplet potential at N^2LO

$$
\begin{aligned}
V_T(r) \;=\; & -\frac{2}{3}\frac{\alpha_s(1/r)}{r}\left\{1+\frac{\alpha_s(1/r)}{4\pi}\left[\frac{31}{3}+22\gamma_E-\left(\frac{10}{9}+\frac{4}{3}\gamma_E\right)n_f\right]\right. \\
& +\left(\frac{\alpha_s(1/r)}{4\pi}\right)^2\left[+66\zeta(3)+484\gamma_E^2+\frac{1976}{3}\gamma_E+\frac{3}{4}\pi^4+\frac{121}{3}\pi^2+\frac{4343}{18}\right. \\
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& \left.\left.+\left(\frac{16}{9}\gamma_E^2+\frac{80}{27}\gamma_E+\frac{4}{27}\pi^2+\frac{100}{81}\right)n_f^2\right]\right\}
\end{aligned}
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OBTAINED BY SENDING A QUARK TO INFINITY

Conclusions

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The complete NNLO QQQ singlet and QQ antitriplet static potential has been calculated

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These results open the way to the study of renormalization group and ultrasoft corrections for the QQQ static energy (as it has been done for the qqbar case) and to the study of the gluelumps for QQQ