Simulations of Plasma Dynamics in HPRF using Code SPACE

Kwangmin Yu
Dept. of Applied Mathematics and Statistics
Stony Brook University, Stony Brook NY 11794

Roman Samulyak
Dept. of Applied Mathematics and Statistics
Stony Brook University, Stony Brook NY 11794
Computational Science Center
Brookhaven National Laboratory, Upton, NY 11973

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Introduction

- HPRF cavity has been proposed for muon cooling.
- An important issue in the cavity is a RF power loading.
- Plasma dynamics relevant to RF power loading has been studied by simulations.
- Computational algorithms of atomic physics processes for the plasma dynamics has been invented and implemented based SPACE (EM-PIC code).
- Simulations support the experimental program on the HPRF cavity in the MTA at Fermilab.
- Simulations suggest ion-electron recombination rate.
- Other plasma properties have been being studied.

Physical Parameters
in Simulations and Experiments

Parameter	Units	Value
Kinetic Energy of Beam	MeV	400
Initial Velocity of Beam	m/s	2.13728e+8
β	%	71.292
H_2 Gas (Mass) Density	g/cm^3	0.00867
H_2 Gas (Number) Density	$\#/cm^3$	2.59e+21
dE/dx	MeV cm^2 / g	6.332
W (Average Ionization Energy)	eV	36.2
Electric Field (Frequency)	MV/m (MHz)	20 (801.6)
Magnetic Field	T	3
Bunch Population	# / bunch	2.23e+8
Bunch Spacing	nanosecond	5
# of Bunches	#	2000

Table 1

Code SPACE

- Developed Electromagnetic PIC code called **SPACE** for particle beams and electromagnetic fields
- Main novelty:
  - Fully relativistic treatment of particles
  - Resolution of atomic physics processes / plasma chemistry
  - Interaction of plasma with neutral matter
  - Advanced numerical solutions
    - Approximations enabling long physical time simulations
    - Adaptive refinement by variable particle mass / charge
    - Data transfer algorithms between relativistic moving and laboratory frames that transform particles to the same physical time
  - Implementation for modern multicore supercomputers
- **Support of BNL RHIC projects**
- Use of plasma for the mitigation of beam-beam effects
- Simulations of Coherent Electron Cooling

Plasma Loading

$$\frac{dV(t)}{dt} = \frac{V_0 - V(t)}{RC} - \frac{P}{CV(t)}$$

V_0: peak voltage, $V(t)$: RF amplitude at t,

$P = \frac{dw}{T} \times \boldsymbol{n}$: power consumption in the cavity,

$R = 1.6\ M\Omega, C = 1.51\ pF,$ (Ref. Ben's thesis, Appendix C.)

$dw = \int_0^T q\,\mu\,(E)^2 \sin(\omega t)dt = \frac{1}{2}\,q\,\mu\,(E)^2\,T$: **average power dump by one ion-electron pair during one time period of the external electric field,**

T: period of external field & $\omega = 2\pi/T$,

E: amplitude of electric field applied to the cavity (Ref. **Appendix.**),

μ : electron mobility, q : charge of electron,

$\boldsymbol{n}$: number density of electrons.

The density of electrons affect to the plasma loading.

Recombination

(Pure Hydrogen)

Recombination Formula

$$\frac{dn_e}{dt} = N - \sum \beta_{H_n{}^+}\, n_e\, n_{H_n{}^+} \quad (1)$$

N = production rate of electrons, $\beta_{H_n{}^+}$ = effective recombination rate

n_e = number density of electrons, $n_{H_n{}^+}$ = number density of $H_n{}^+$

- There are higher $H_n{}^+$ $(n > 3)$ clusters.
- Different hydrogen ion clusters have different recombination rates.
- The recombination rates of higher hydrogen ion clusters are unknown.
- We don't know the component ratio of hydrogen ion clusters.

$$\frac{dn_e}{dt} = N - \beta_e\, n_e \sum n_{H_n{}^+} \quad (2)$$

- **Instead of (1), equation (2) and $\boldsymbol{\beta_e}$ are used where $\boldsymbol{\beta_e}$ is the effective recombination rate measured in HPRF cavity.**
- **$\boldsymbol{\beta_e}$ is unknown except at the equilibrium status.**

Recombination Rate (β)

$\boldsymbol{\beta = a\,X^b}$ where $X = E/P$ (MV/m/psi)

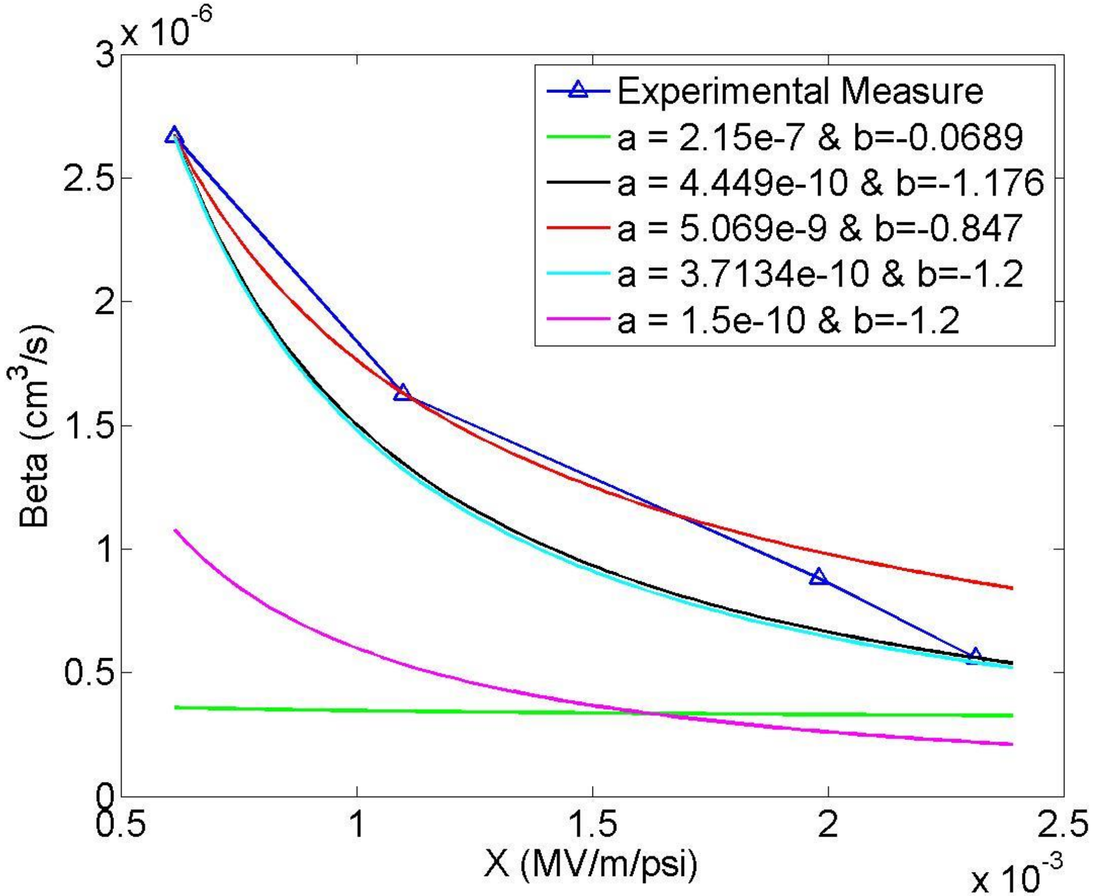
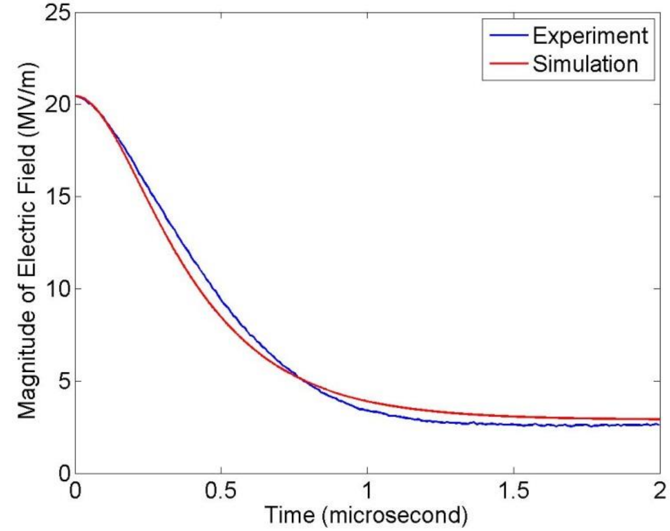


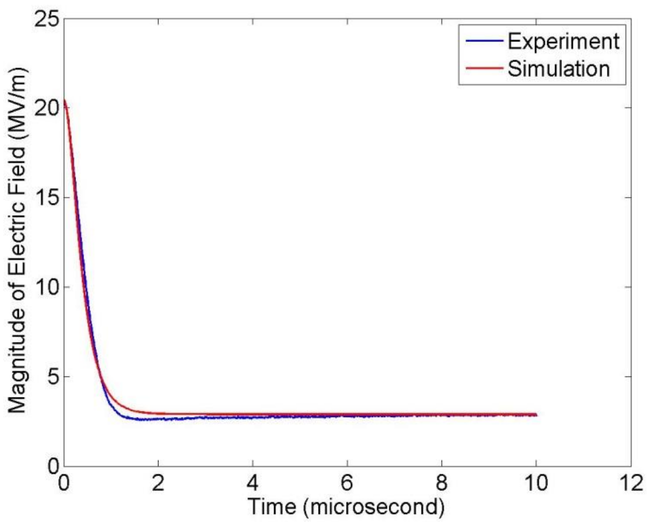
Figure. Recombination curves in 1470 psi (100 atm)

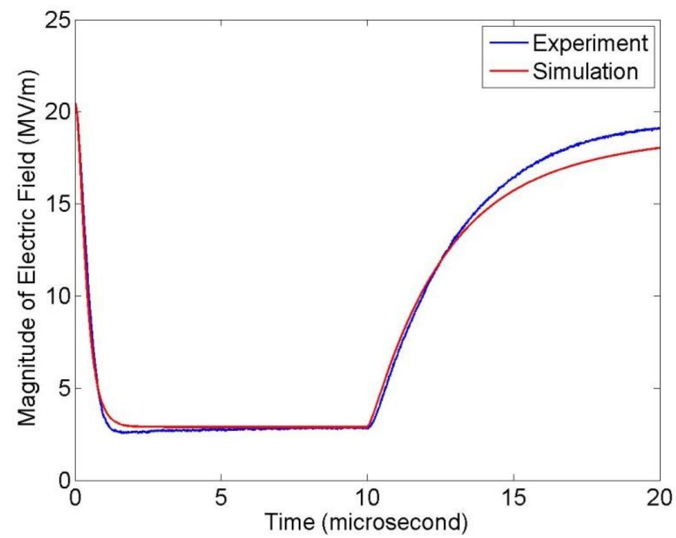
- There are only 4 data in 1470 psi and high density beam experiment.
- Based on the data, other parameters are tested.

Simulation Result
(Electric Field Intensity)

- Various coefficients were tested.
- The most accurate combination is $\boldsymbol{\beta = (1.5e - 10\ )X^{-1.2}}$.
- Beam off at 10 μs.
- Three figures with different time scales.



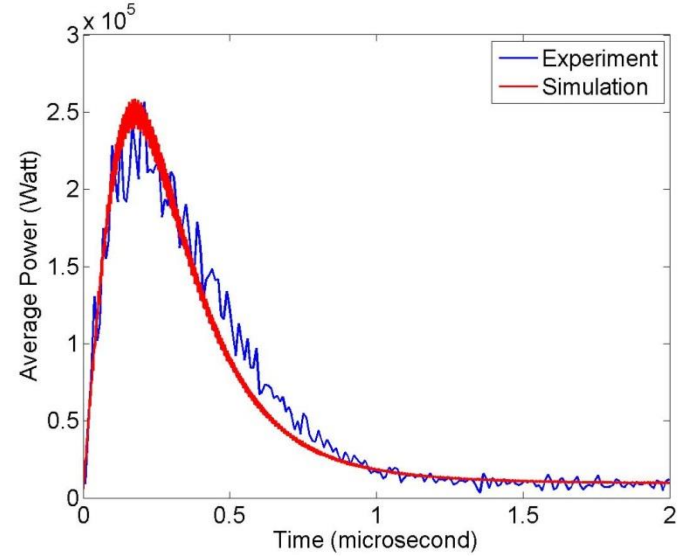


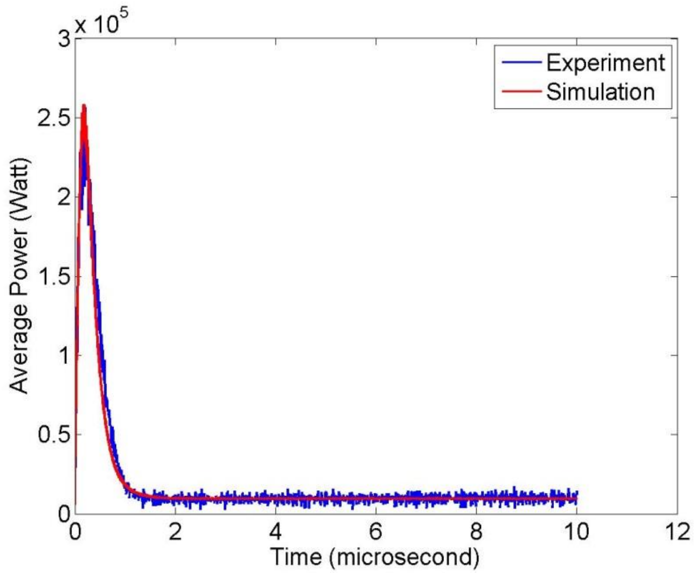


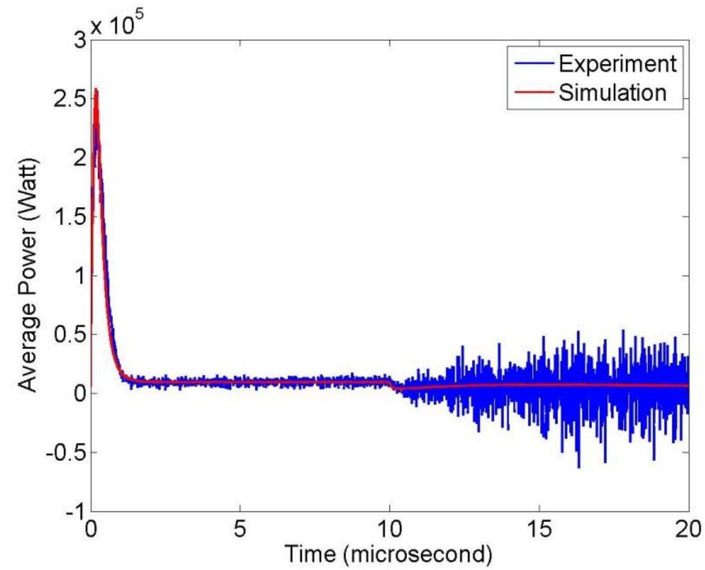
Simulation Result

(Power Dump by Plasma)

- $\boldsymbol{\beta = (1.5e-10)X^{-1.2}}$.
- Beam off at 10 μs.
- The reason of thick red band is the oscillation of value in short period.
- External electric field period is about 1.25 ns.

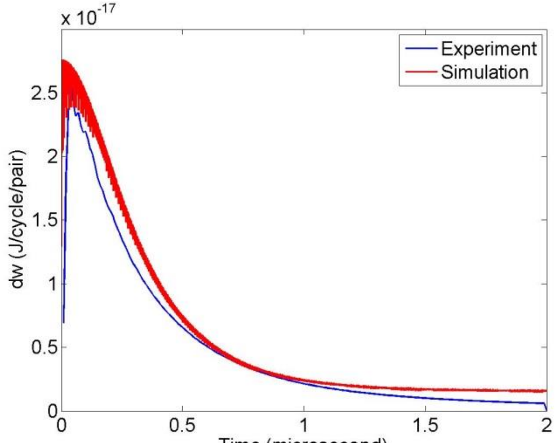


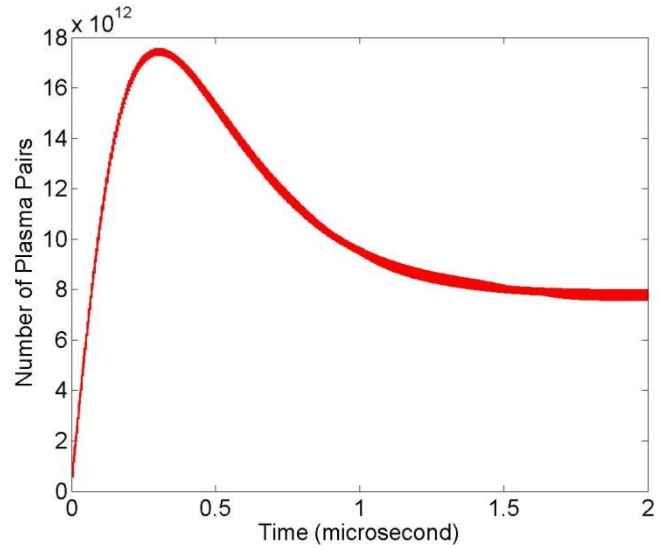


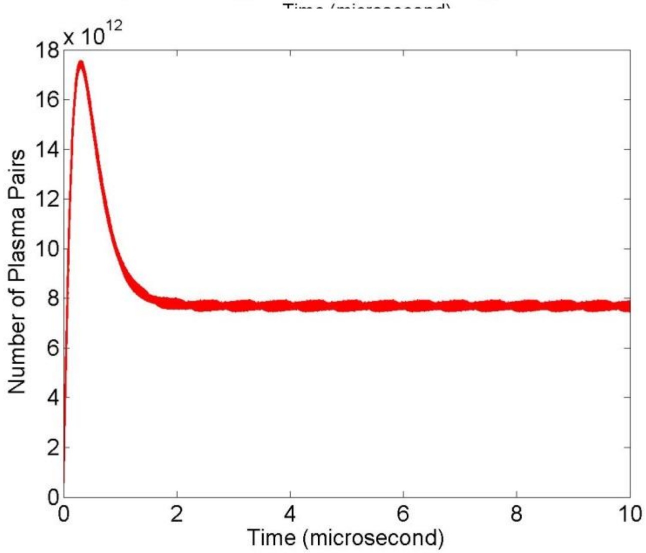


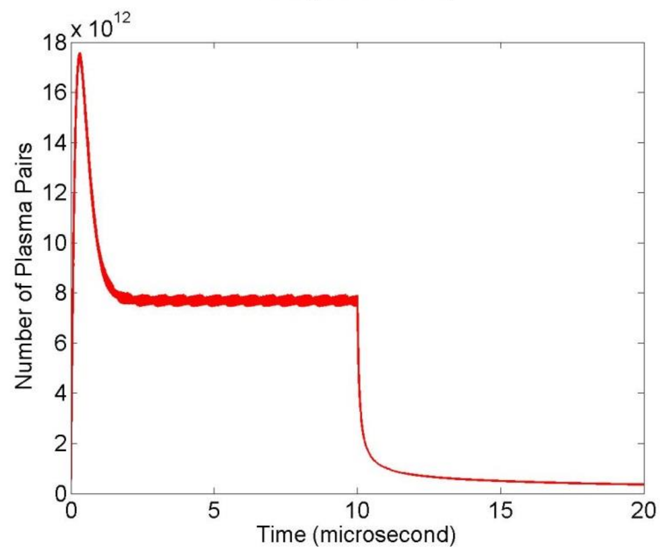
Simulation Result
(dw & number of electrons)

- $\boldsymbol{\beta = (1.5e-10)X^{-1.2}}$.









Attachment

(With Dry Air Dopant)

Recombination & Attachment Formulae

$$\frac{dn_e}{dt} = N - \beta_e\, n_e \sum n_{H_n{}^+} \tag{2}$$

From the equation (2), electron attachment term on dopants are added.

$$\frac{dn_e}{dt} = N - \beta_e\, n_e n_{H^+} - \frac{n_e}{\boldsymbol{\tau}} \tag{3}$$

$$\frac{dn_{H^+}}{dt} = N - \beta_e\, n_e n_{H^+} - \boldsymbol{\eta}\, n_{H^+} n_{O_2{}^-} \tag{4}$$

$$\frac{dn_{O_2{}^-}}{dt} = \frac{n_e}{\boldsymbol{\tau}} - \boldsymbol{\eta}\, n_{H^+} n_{O_2{}^-} \tag{5}$$

where $n_{H^+} = \sum n_{H_n{}^+}$: the sum of all hydrogen ion cluster numbers,

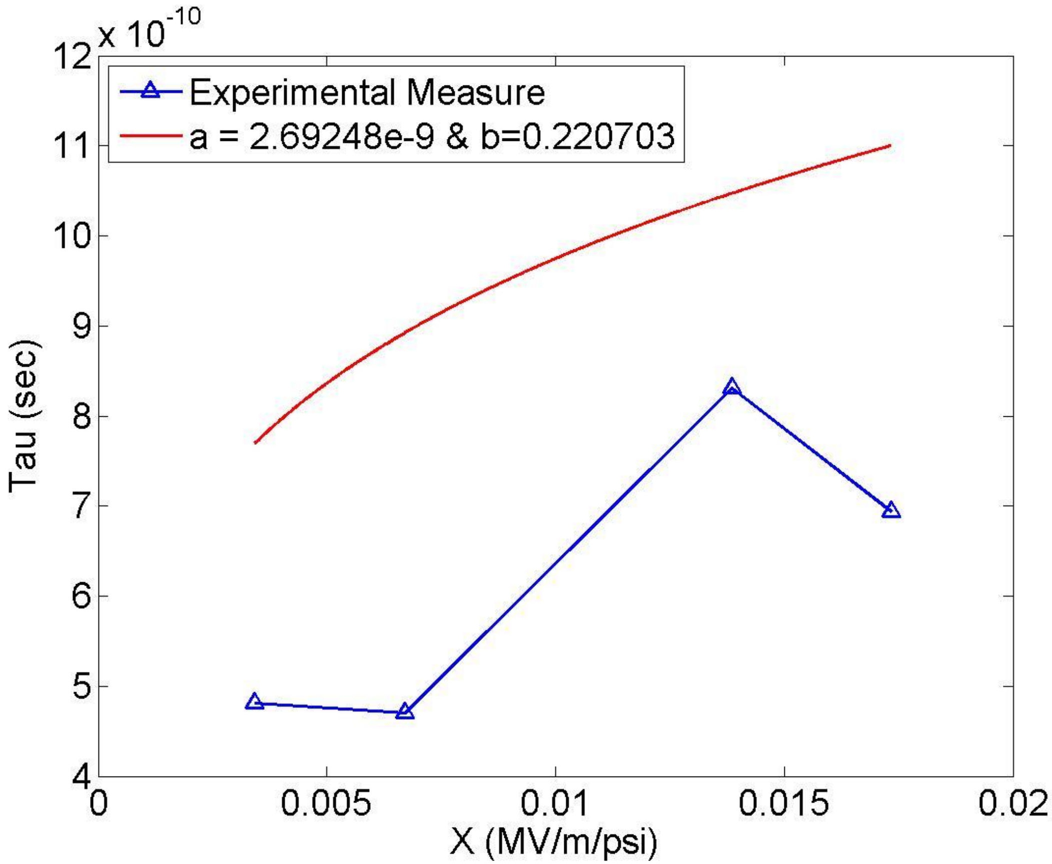
$\boldsymbol{\eta}$: the effective recombination rate of hydrogen ion clusters and oxygen ion,

$\boldsymbol{\tau}$ is the attachment time of electrons on the dopant molecules.

$\frac{1}{\boldsymbol{\tau}} = \sum_m k_m n_{O_2} n_m$, m is one of $H_2, O_2, or\ N_2$,

Attachment Time (τ)

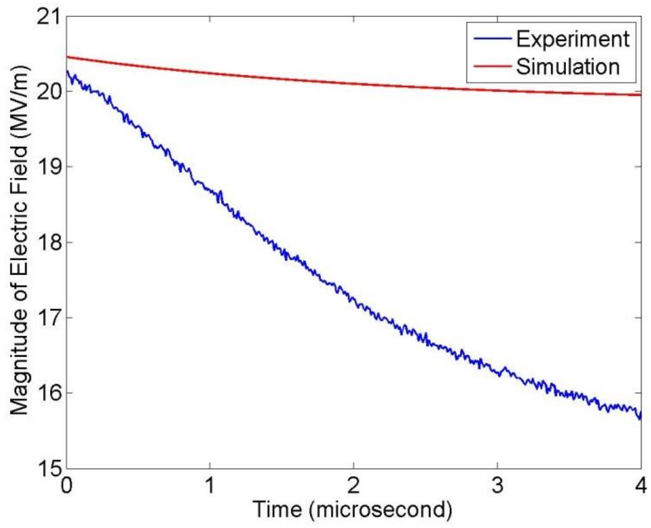
$\boldsymbol{\tau = a\,X^b}$ where $X = E/P$ (MV/m/psi)

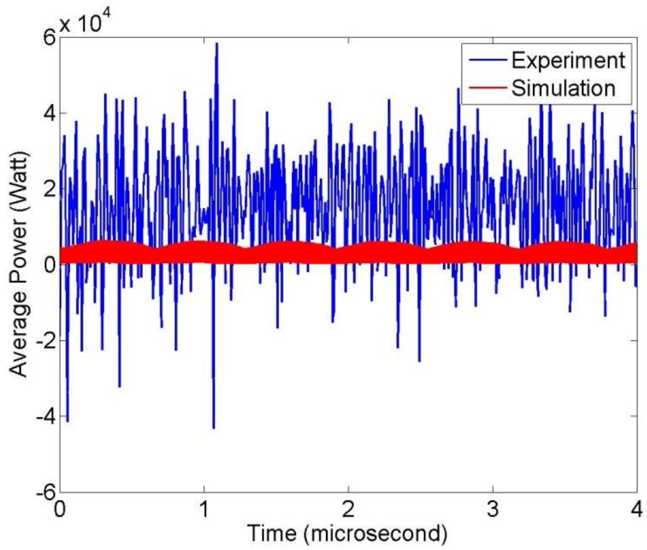


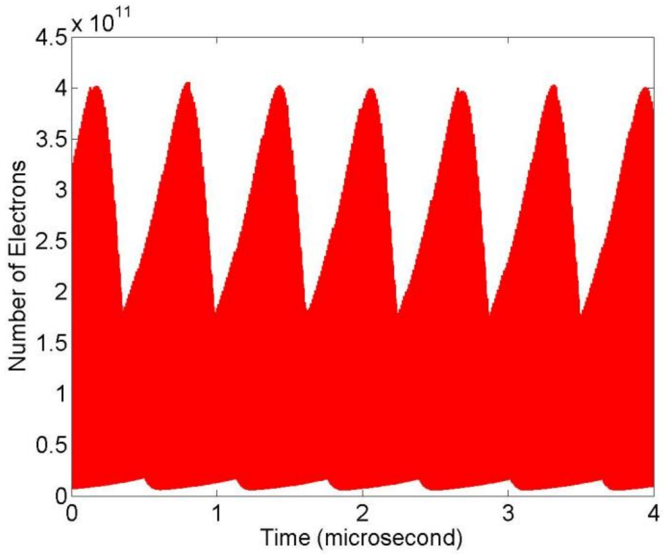
- There are only 4 data in 1470 psi and high density beam experiment doped with **1 % dry air**.
- Based on the data, the parameter is tested.

Preliminary Simulation (1)

- $\beta = (1.5e-10\,)X^{-1.2}$
- $\tau = (2.69e-9)X^{0.22}$
- Ion recombination rate
  $\eta = (6.32e-9)X^{-0.34}$
- η is chosen from IPAC14, THPRI064.
- Results are far from experimental data.
- Other τ is tested.

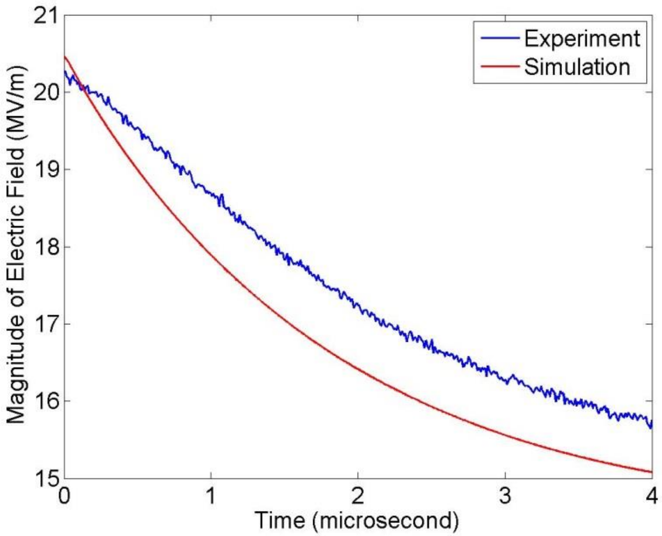


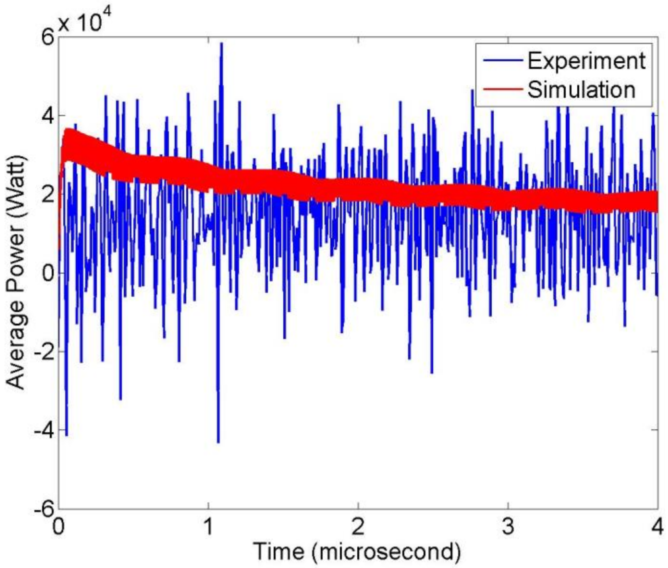


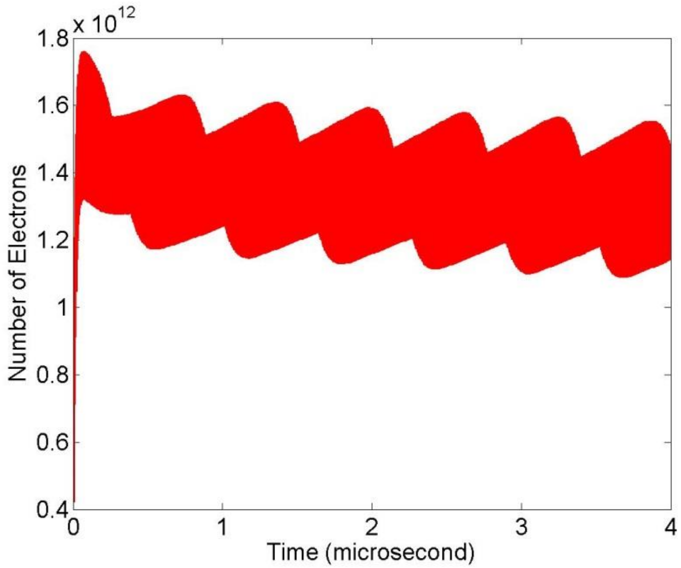


Preliminary Simulation (2)

- Different τ is tested.
- $\beta = (1.5e - 10)X^{-1.2}$
- $\tau = (1.35e - 8)X^{0.0}$
- $\eta = (6.32e - 9)X^{-0.34}$
- The simulation result is similar to the experimental data.
- Other τ is tested.

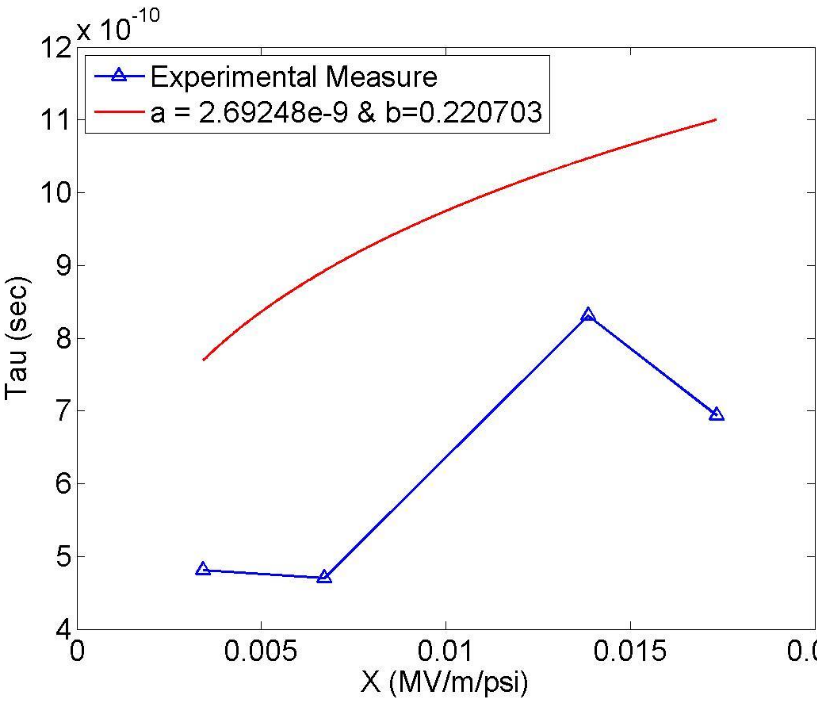


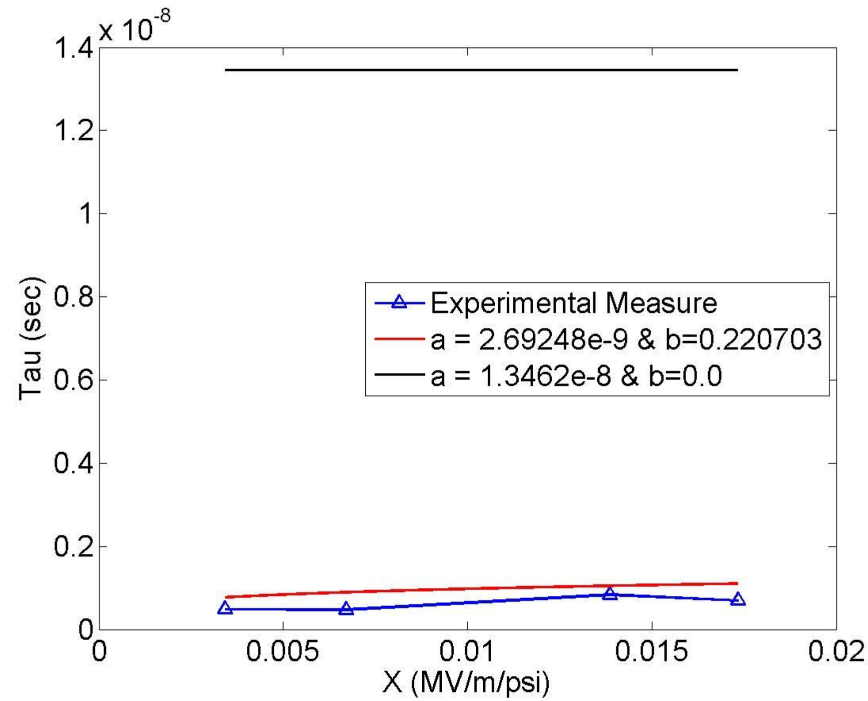




Attachment Time (τ)

$\tau = a\,X^b$ where $X = E/P$ (MV/m/psi)





- New attachment time $\boldsymbol{\tau = (1.35e-8)X^{0.0}}$ is far from the experimental measurement of τ, although the result is similar to the experimental data when the $\boldsymbol{\tau}$ is used.
- Ion-ion recombination rates should be tested.

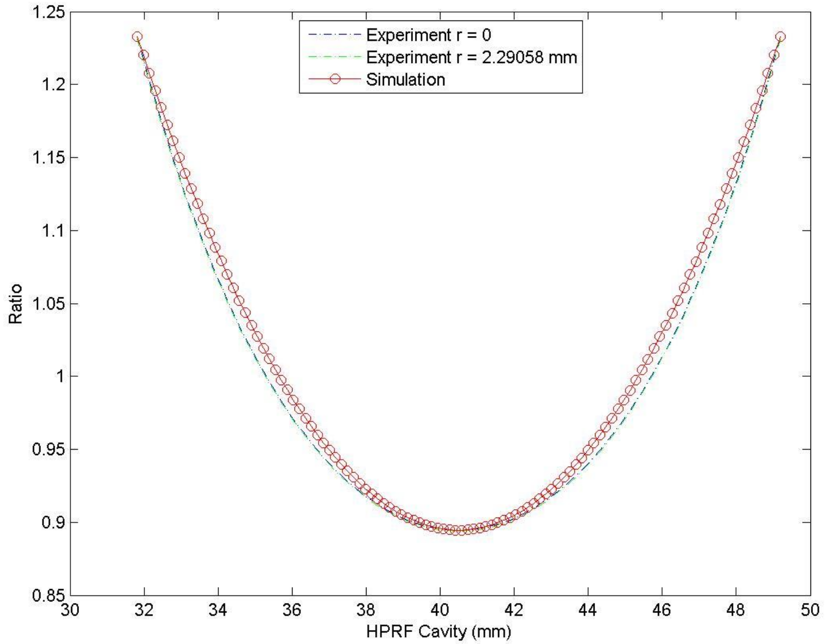
Conclusion

I. Simulations suggest a very accurate fitting function for beta. ($\beta = \mathbf{1.5e-10}\ \boldsymbol{X}^{-\mathbf{1.2}}$)

II. More research and simulations are needed to obtain more accurate attachment time (τ) and ion – ion recombination rate (η).

III. These plasma properties will be used for much denser muon beams.

Appendix



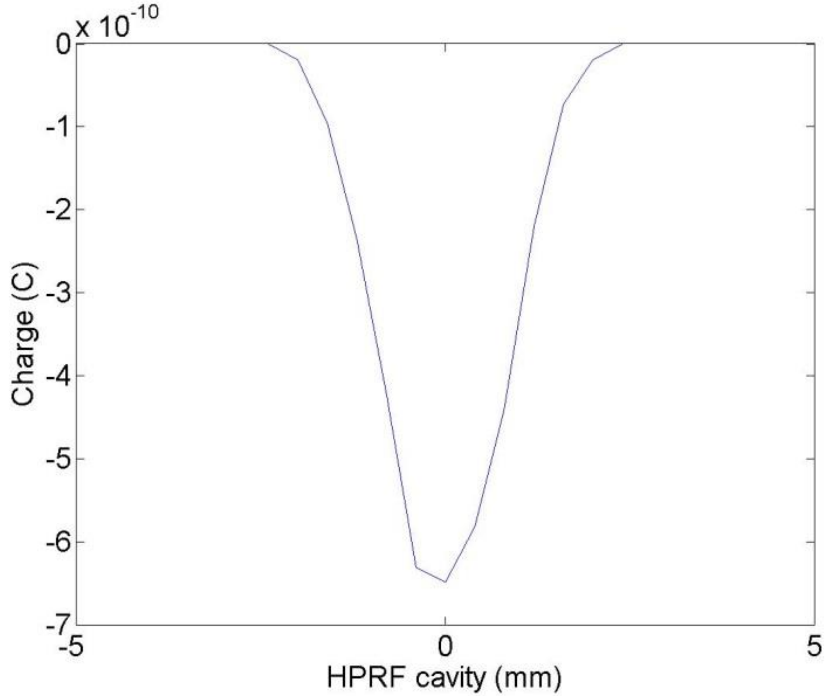


Fig. External Field Profile in HPRF cavity
Appr. Formula :
Ratio = 0.004484 z^2 −0.3632 z + 8.249

Fig. Transversal distribution
of electrons in the cavity center
at 0.185 ns