Spin dynamics of interacting many-particle quantum systems

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S.K. Haldar, N.D. Chavda, Manan Vyas, V.K.B. Kota, J. Stat. Mech. (2016) 043101

Outline

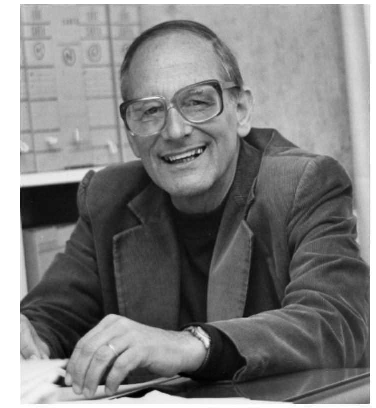
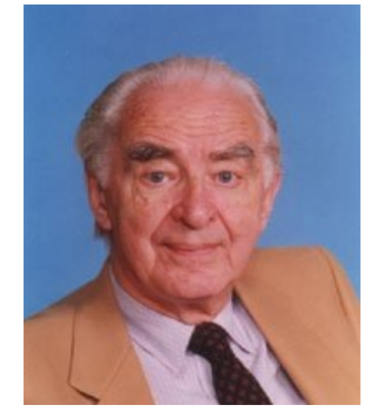
- Model EE(1+2)-S
- Chaos markers
- Quench dynamics
- Fidelity
- Information entropy
- Conclusions

Model EE(1+2)-S

Embedded Random Matrix Ensembles





Wishart (1928) Wigner (1955) Mehta (1965) Dyson (1962) Brody (1970) French (1970)

"The assumption is that the Hamiltonian which governs the behavior of a complicated system is a random symmetric matrix, with no special properties except for its symmetric nature." E.P. Wigner

Depending on the global symmetry properties, namely rotational and time-reversal, the classical random matrix ensembles are classified into three classes- Gaussian orthogonal (GOE), unitary (GUE) and symplectic (GSE) ensembles. The corresponding matrices will be real symmetric, complex hermitian and real quaternion matrices. (Cartan, Dyson)

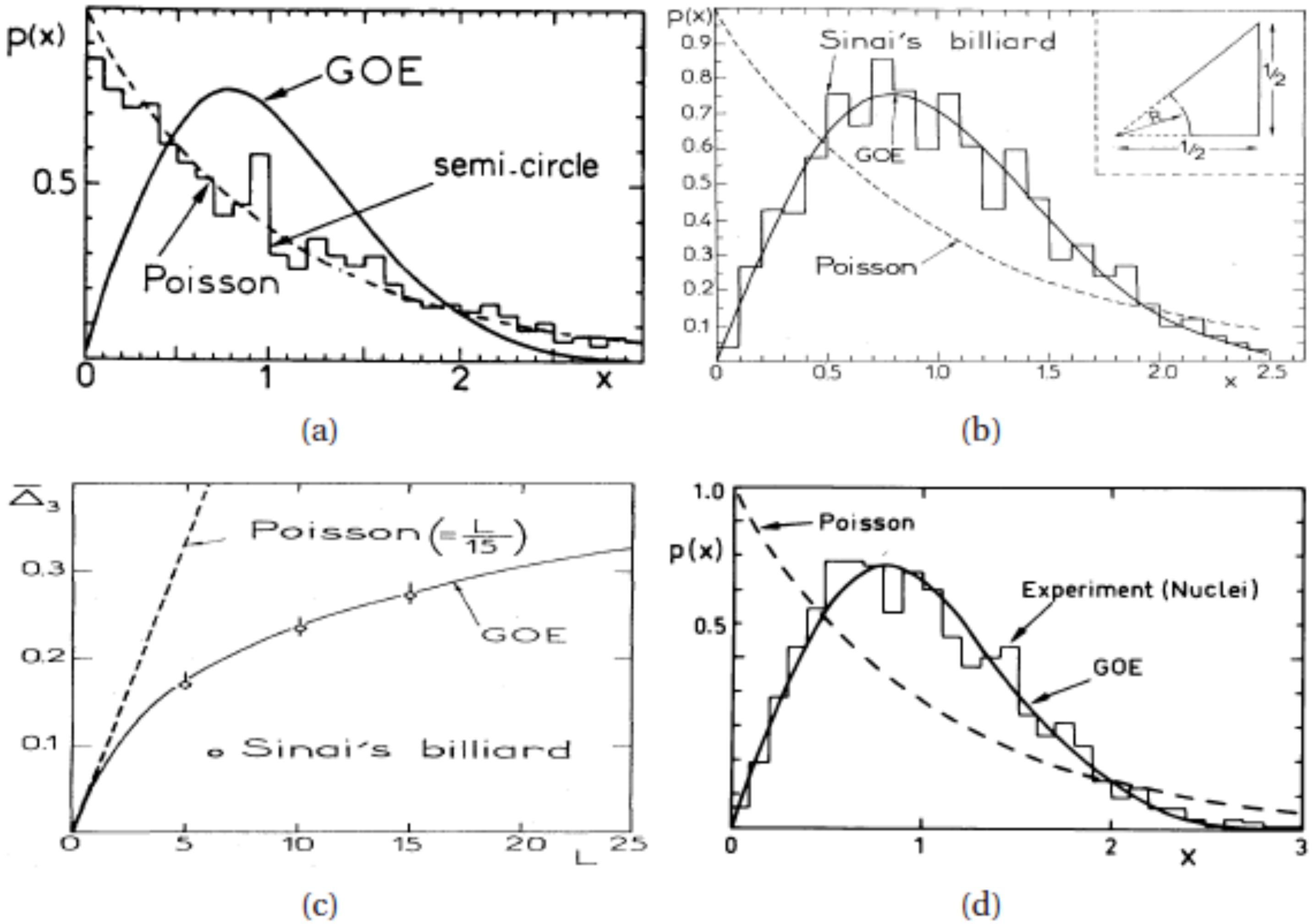


Figure illustrating the connection between RMT and quantum chaos.

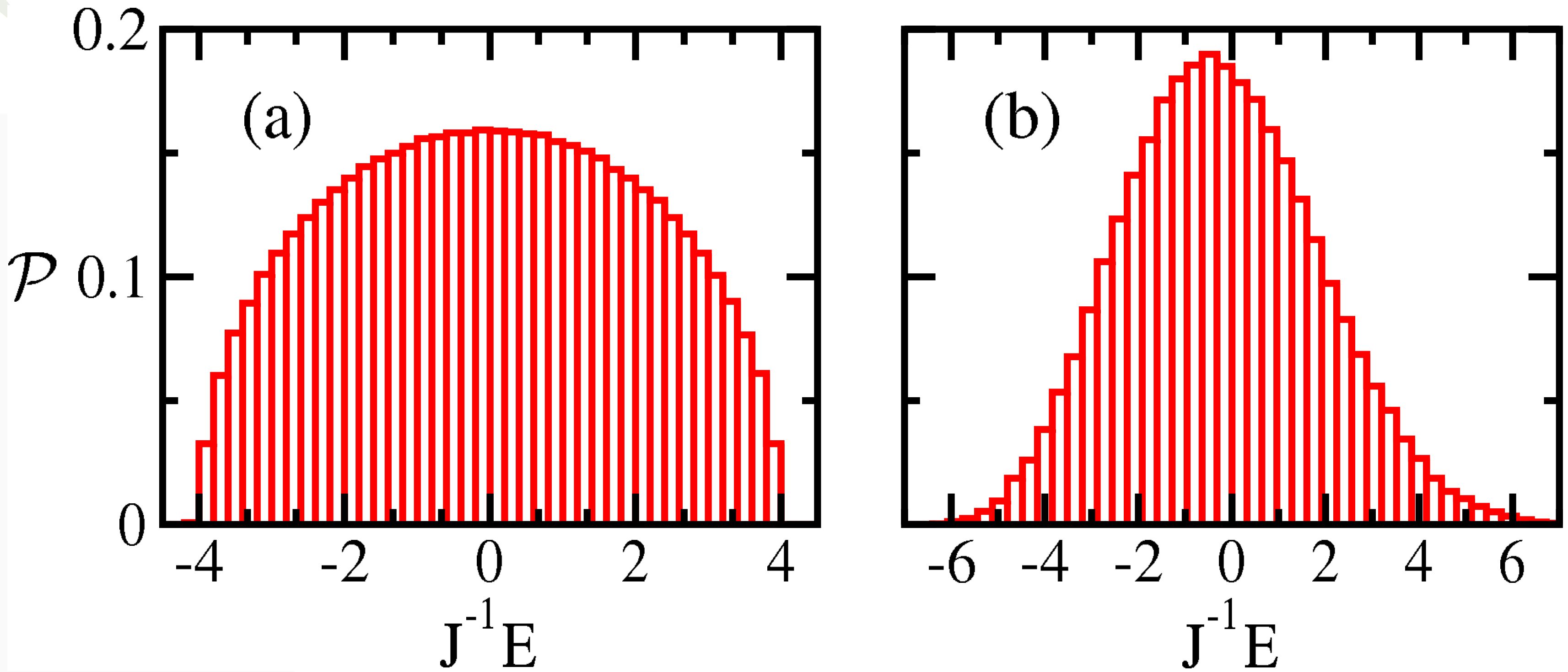
"Classical dynamical systems can be separated into two classes - integrable and chaotic. For quantum systems this distinction manifests itself, e.g. in spectral statistics. Roughly speaking integrability leads to Poisson distribution for the energies (M.V. Berry) while chaos implies Wigner-Dyson statistics of levels (O. Bohigas), which are characteristic for the ensemble of random matrices. The onset of chaotic behavior for a rather broad class of systems can be understood as a delocalization in the space of quantum numbers that characterize the original integrable system ……" **Altshuler (2004)**

"We speak of chaos in quantum systems if the statistical properties of the eigenvalue spectrum coincide with predictions of random-matrix theory. Chaos is a typical feature of atomic nuclei and other self bound Fermi systems " Papenbrock and Weidenmueller (2007)

The GOE, now almost universally regarded as a model for a corresponding chaotic system is an ensemble of multi-body, not two-body interactions. This difference shows up both in one-point (density of states) and two-point (fluctuations) functions.

French and Wong (1970), Bohigas and Flores (1971), Benet and Weidenmueller (2003)

State Density:



E. J. Torres-Herrera, Manan Vyas and L. F. Santos, New Journal of Physics 16, 063010/1-33 (2014)

EGOE(2)[BEGOE(2)] for fermion[boson] systems

N single particle states, m fermions[bosons]

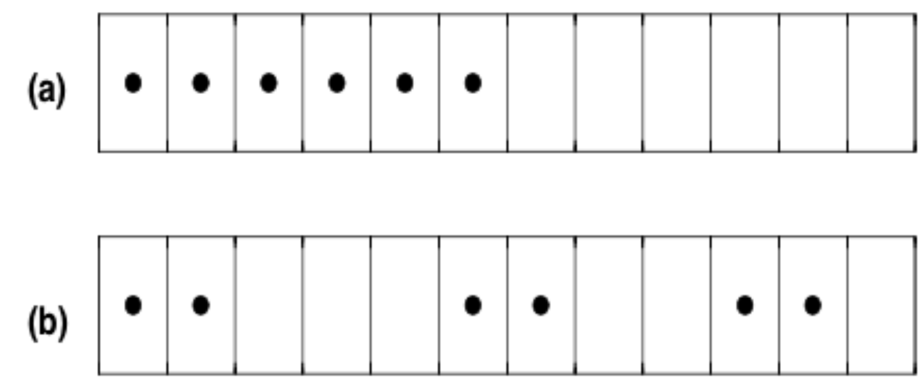
fermions:

$$\widehat{H} = \sum_{\nu_i<\nu_j,\,\nu_k<\nu_\ell} \langle \nu_k\, \nu_\ell \mid \widehat{H} \mid \nu_i\, \nu_j \rangle\, a^\dagger_{\nu_\ell}\, a^\dagger_{\nu_k}\, a_{\nu_i}\, a_{\nu_j}$$

Number of basis states: $d(N,m) = \binom{N}{m}$

d(12,6)=924, d(16,8)=12870

Basis states: N=12, m=6



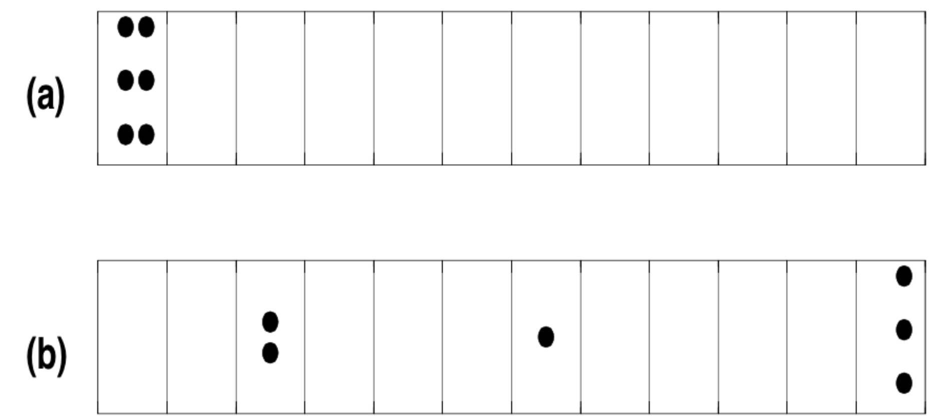
bosons:

$$\widehat{H} = \sum_{\nu_i\le\nu_j,\,\nu_k\le\nu_l} \frac{\langle \nu_k\, \nu_l \mid \widehat{H} \mid \nu_i\, \nu_j \rangle}{\sqrt{(1+\delta_{ij})(1+\delta_{kl})}}\, b^\dagger_{\nu_k}\, b^\dagger_{\nu_l}\, b_{\nu_i}\, b_{\nu_j}$$

Number of basis states: $d(N,m) = \binom{N+m-1}{m}$

d(12,4)=1365 , d(12,6)=12376, d(16,8)=490314

Basis states: N=12, m=6



$\langle \nu_k\, \nu_\ell \mid \widehat{H} \mid \nu_i\, \nu_j \rangle$ are independent Gaussian random variables

$$\overline{\langle \nu_k\, \nu_\ell \mid \widehat{H} \mid \nu_i\, \nu_j \rangle} = 0 \qquad \overline{\left|\langle \nu_k\, \nu_\ell \mid \widehat{H} \mid \nu_i\, \nu_j \rangle\right|^2} = v^2 \left(1 + \delta_{(ij),(k\ell)}\right)$$

fermions:

$$\langle \nu_k \nu_\ell | \widehat{H} | \nu_j \nu_i \rangle = -\langle \nu_k \nu_\ell | \widehat{H} | \nu_i \nu_j \rangle$$

$$\langle \nu_k \nu_\ell | \widehat{H} | \nu_i \nu_j \rangle = \langle \nu_i \nu_j | \widehat{H} | \nu_k \nu_\ell \rangle \, .$$

bosons:

$$\langle \nu_k \nu_l | \widehat{H} | \nu_j \nu_i \rangle = \langle \nu_k \nu_l | \widehat{H} | \nu_i \nu_j \rangle$$

$$\langle \nu_k \nu_l | \widehat{H} | \nu_i \nu_j \rangle = \langle \nu_i \nu_j | \widehat{H} | \nu_k \nu_l \rangle$$

- **H matrix in two-particle spaces is GOE**
- **Geometry gives *H* matrix in *m*-particle spaces**
- **Many *m*-particle matrix elements are zero**
- **There are correlations between *m*-particle matrix elements**
- **Easy to extend EE(2) to EE(*k*)**

$k = m \longrightarrow$ GOE

K.K. Mon and J.B. French, Ann. Phys. (N.Y.) 95, 90 (1975).

many-particle matrix elements:

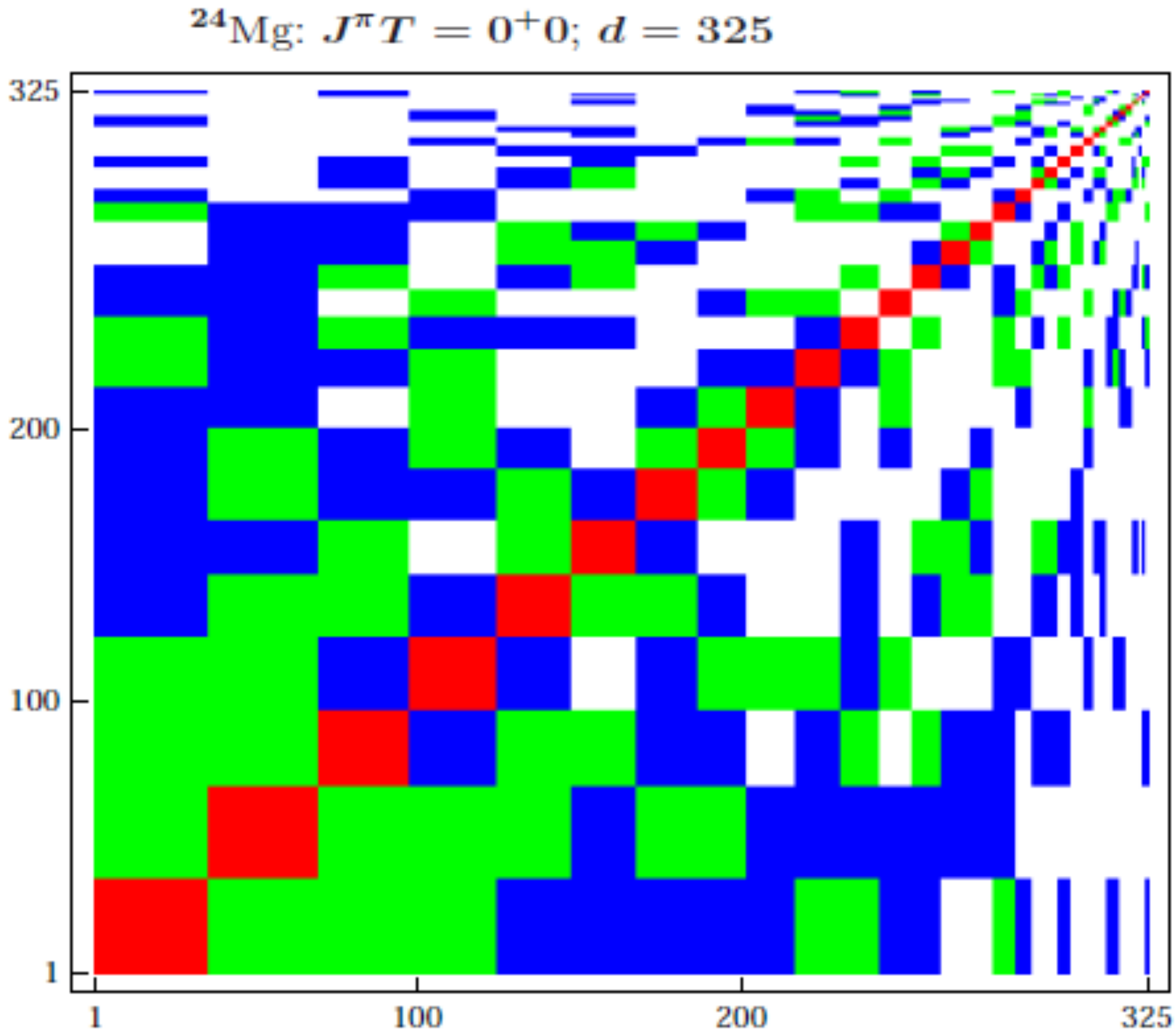
$$\left\langle \prod_{r=i,j,\dots} (\nu_r)^{n_r} \,|\, \hat{H} \,|\, \prod_{r=i,j,\dots} (\nu_r)^{n_r} \right\rangle = \sum_{i\geq j} \frac{n_i\left(n_j-\delta_{ij}\right)}{\left(1+\delta_{ij}\right)} \left\langle \nu_i \nu_j \,|\, \hat{H} \,|\, \nu_i \nu_j \right\rangle ,$$

$$\left\langle (\nu_i)^{n_i-1} \left(\nu_j\right)^{n_j+1} \prod_{r'=k,l,\dots} (\nu_{r'})^{n_{r'}} \,|\, \hat{H} \,|\, \prod_{r=i,j,\dots} (\nu_r)^{n_r} \right\rangle =$$

$$\sum_{k'} \left[\frac{n_i\left(n_j+1\right)\left(n_{k'}-\delta_{k'i}\right)^2}{\left(1+\delta_{k'i}\right)\left(1+\delta_{k'j}\right)} \right]^{1/2} \left\langle \nu_{k'} \nu_j \,|\, \hat{H} \,|\, \nu_{k'} \nu_i \right\rangle ,$$

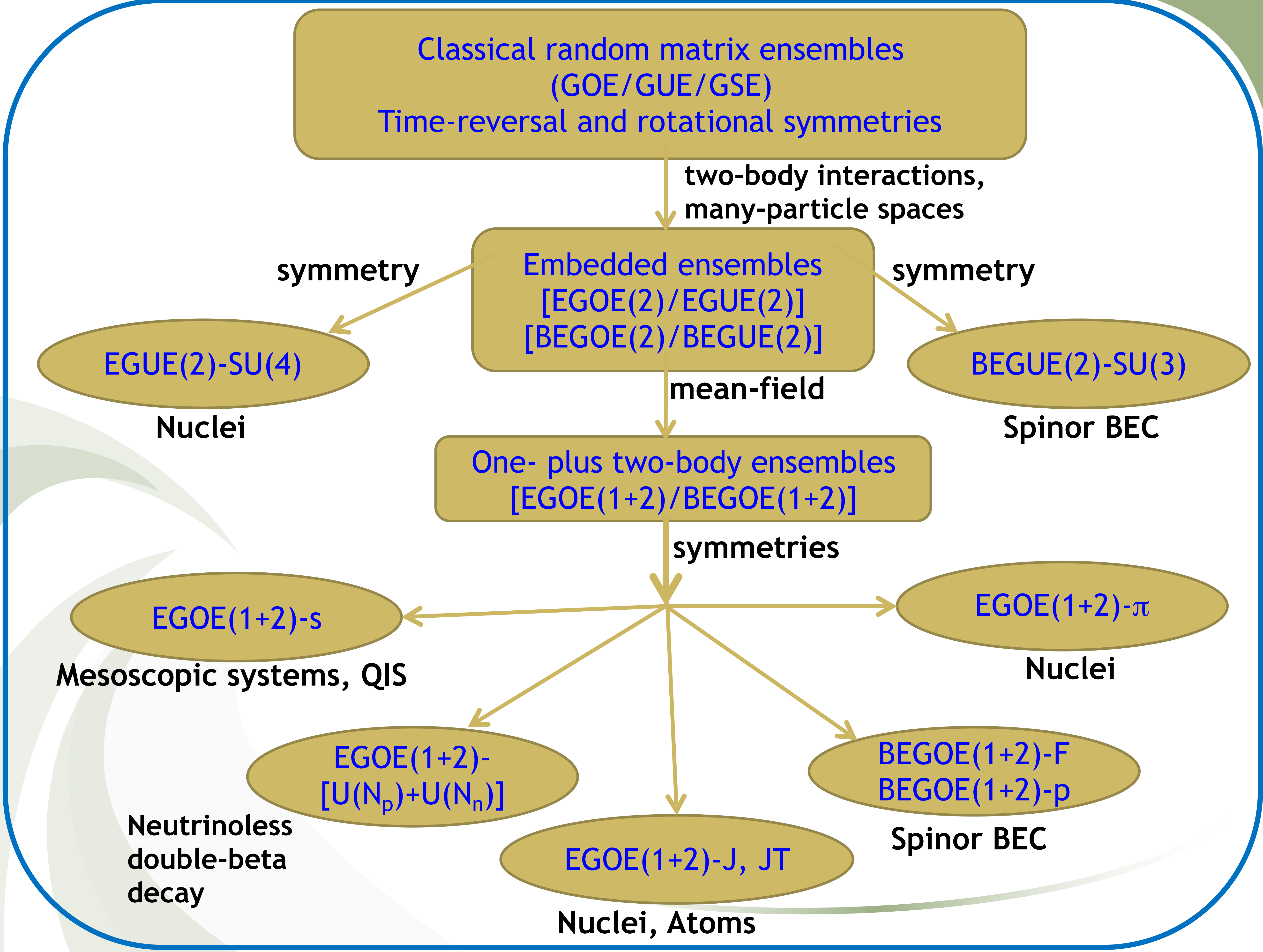
$$\left\langle (\nu_i)^{n_i+1} \left(\nu_j\right)^{n_j+1} (\nu_k)^{n_k-1} (\nu_l)^{n_l-1} \prod_{r'=m,n,\dots} (\nu_{r'})^{n_{r'}} \,|\, \hat{H} \,|\, \prod_{r=i,j,\dots} (\nu_r)^{n_r} \right\rangle =$$

$$\left[\frac{n_k\left(n_l-\delta_{kl}\right)\left(n_i+1\right)\left(n_j+1+\delta_{ij}\right)}{\left(1+\delta_{ij}\right)\left(1+\delta_{kl}\right)} \right]^{1/2} \left\langle \nu_i \nu_j \,|\, \hat{H} \,|\, \nu_k \nu_l \right\rangle .$$



Block matrix structure of the H matrix of ^{24}Mg displaying two-body selection rules. Total number of blocks are 33, each labeled by the spherical configurations (m_1, m_2, m_3). The diagonal blocks are shown in red and within these blocks there will be no change in the occupancy of the nucleons in the three *sd* orbits. Green corresponds to the region (in the matrix) connected by the two-body interaction that involve change of occupancy of one nucleon. Similarly, blue corresponds to change of occupancy of two nucleons. Finally, white correspond to the region forbidden by the two-body selection rules.

Manan Vyas and VKB Kota, EPJA (2010)



Classical random matrix ensembles (GOE/GUE/GSE) Time-reversal and rotational symmetries
two-body interactions, many-particle spaces
Embedded ensembles [EGOE(2)/EGUE(2)] [BEGOE(2)/BEGUE(2)]
symmetry
symmetry
EGUE(2)-SU(4)
Nuclei
BEGUE(2)-SU(3)
Spinor BEC
mean-field
One- plus two-body ensembles [EGOE(1+2)/BEGOE(1+2)]
symmetries
EGOE(1+2)-s
Mesoscopic systems, QIS
EGOE(1+2)-π
Nuclei
EGOE(1+2)-[U(Np)+U(Nn)]
Neutrinoless double-beta decay
EGOE(1+2)-J, JT
Nuclei, Atoms
BEGOE(1+2)-F BEGOE(1+2)-p
Spinor BEC

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Embedded Ensembles with spin

$$\widehat{H} = \widehat{h}(1) + \lambda\,\widehat{V}(2)$$

$$\widehat{h}(1) = \sum_{i=1,2,\ldots,\Omega} \epsilon_i\, n_i$$

$$V^{s}_{ijkl} = \langle (kl)s, m_s | \widehat{V}(2) | (ij)s, m_s \rangle$$

$$\widehat{V}(2) = \lambda_0\, \widehat{V}^{s=0}(2) + \lambda_1\, \widehat{V}^{s=1}(2)$$

H matrix dimensions are:

$d_f(8,8,S)$=1764, 2352, 720 for spin S=0-2;
$d_f(12,12,S)$=226512, 382239, 196625 for spin S=0-2;

$d_b(4,11,F)$=504, 900, 1100 for F=1/2-5/2;
$d_b(6,12,F)$=13860, 37422, 50050 for F=0-2;

Parameters in the model are (Ω, m, S, λ)

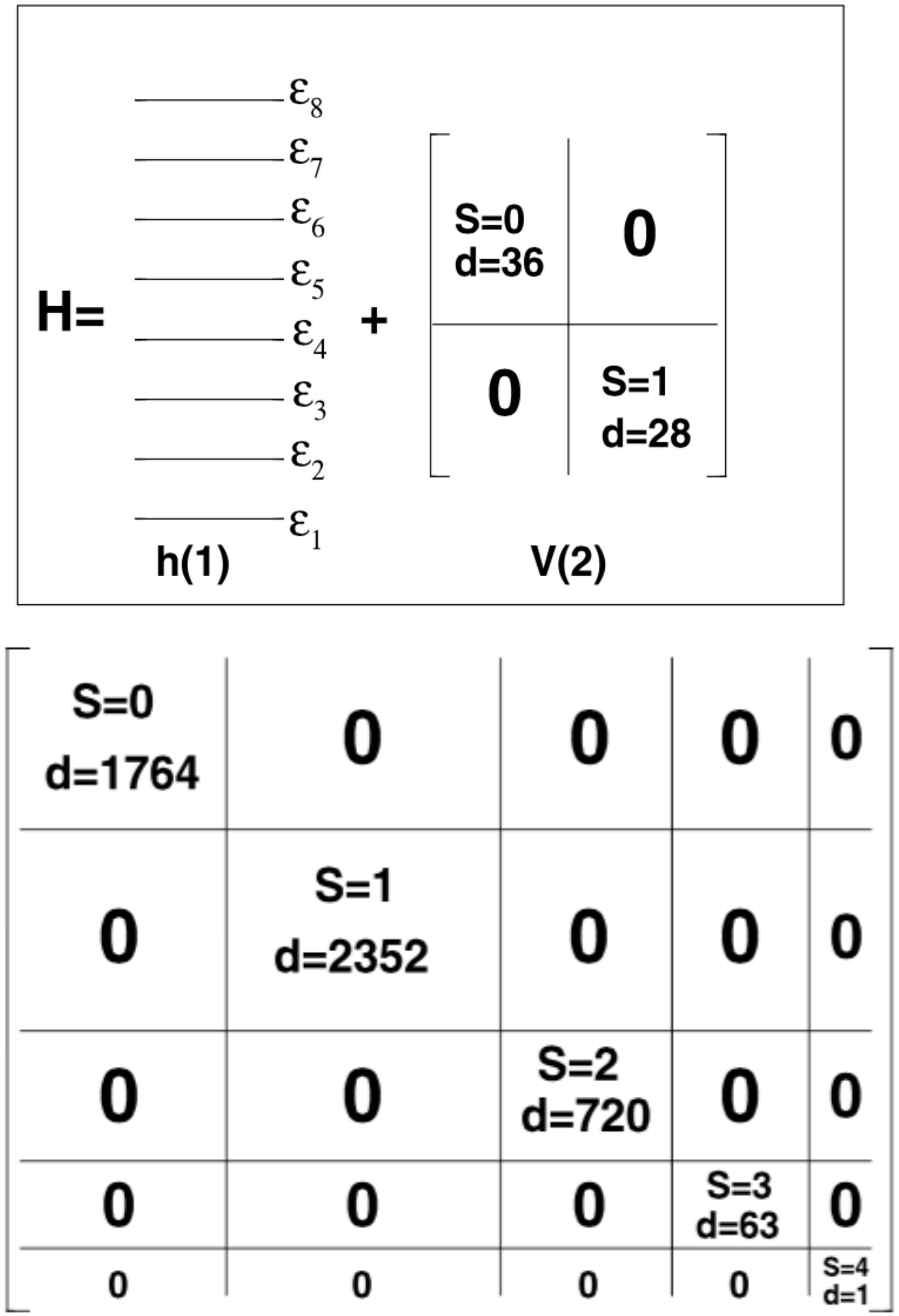
S = m/2, m/2-1, …, 0 or 1/2

ϵ_i : fixed/random/from GOE
Average spacing Δ of single particle levels is chosen to be unity
$V^{s=0,1}$ are independent GOE(0,1) in two-particle spaces

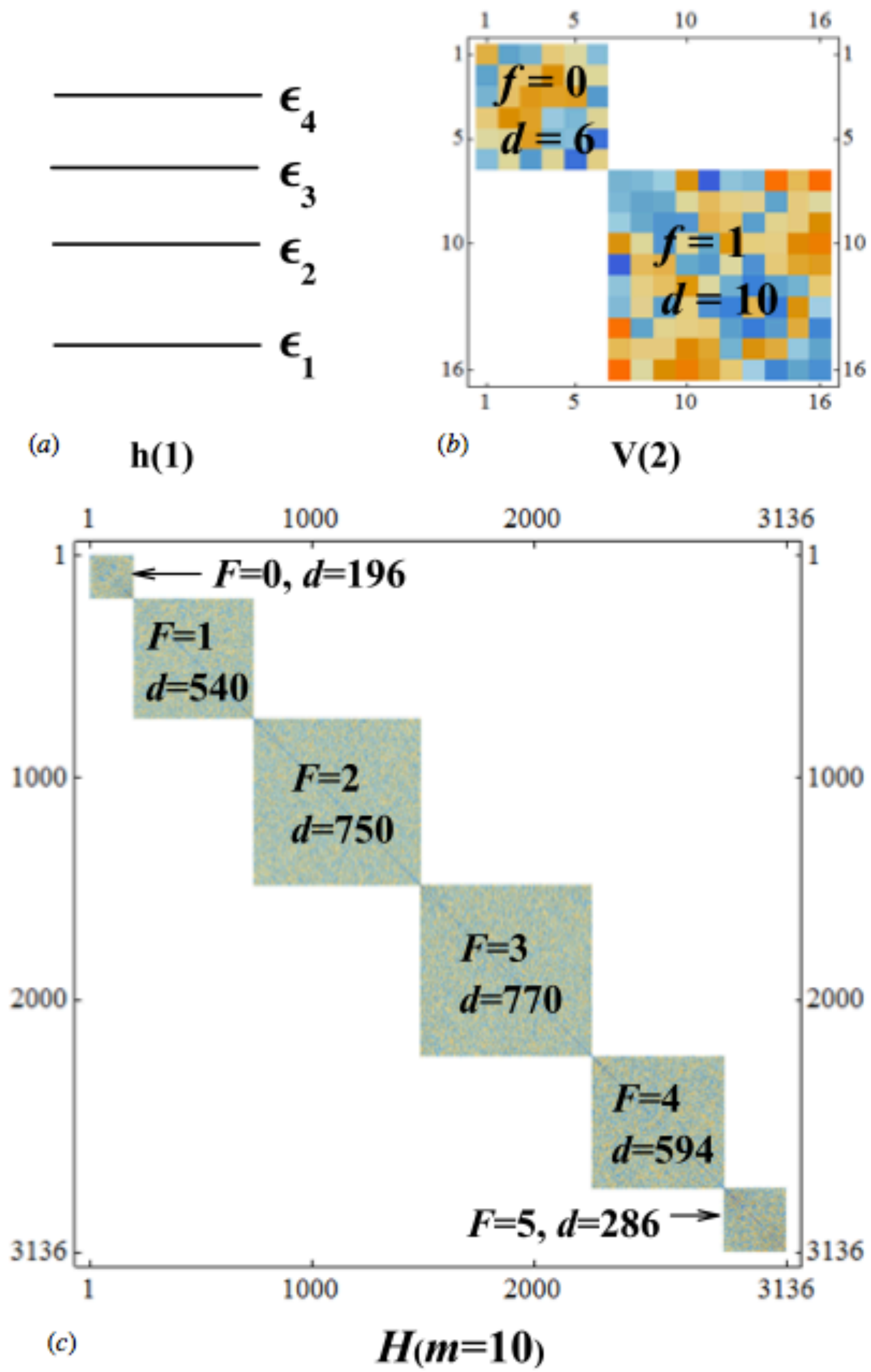
Basis:

Manan Vyas, V.K.B. Kota and N.D. Chavda, Phys. Rev. E 81, 036212/1-17 (2010)
Manan Vyas, N.D. Chavda, V.K.B. Kota and V. Potbhare, J. Phys. A: Math. Theor. 45, 265203 (2012)

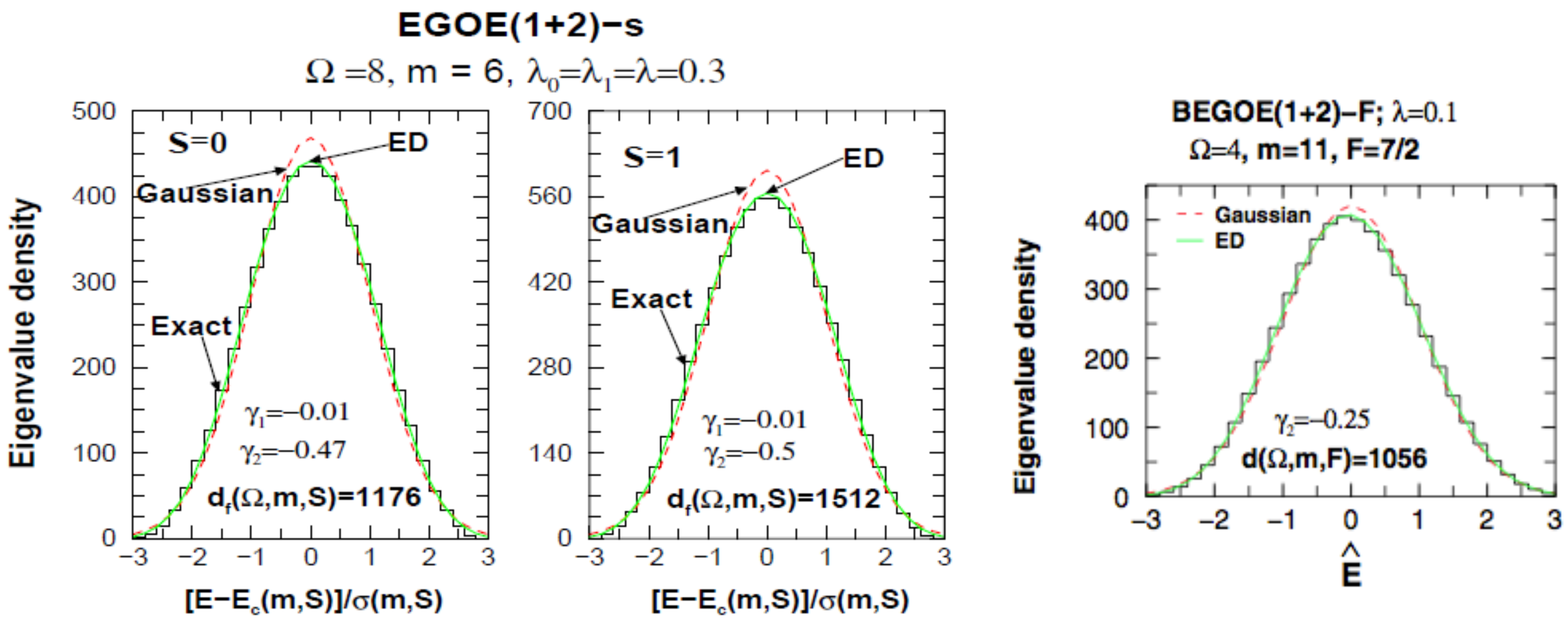
EGOE(1+2)–s : Ω=m=8



BEGOE(1+2)-*F* : Ω=4, *m*=10



Fixed-*(m,S)* state densities: $\rho^{m,S}(E) = \langle \delta(H-E) \rangle^{m,S}$



$$\rho_{\mathcal{G}}(\hat{E}) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\hat{E}^2}{2}\right),$$

$$\rho_{ED}(\hat{E}) = \rho_{\mathcal{G}}(\hat{E}) \left\{ 1 + \left[\frac{\gamma_1}{6} \mathrm{He}_3(\hat{E}) \right] + \left[\frac{\gamma_2}{24} \mathrm{He}_4(\hat{E}) + \frac{\gamma_1^2}{72} \mathrm{He}_6(\hat{E}) \right] \right\}.$$

Ensemble averaged spectral variances

$$\sigma^2(m,S)=\Sigma_{p=0}^{4}a_p m^p+\Sigma_{q=0}^{2}b_q m^q S(S+1)+c_0[S(S+1)]^2$$

$$\overline{\sigma^2_{V(2)}(\Omega,m,S)}=\lambda^2 P(\Omega,m,S)$$

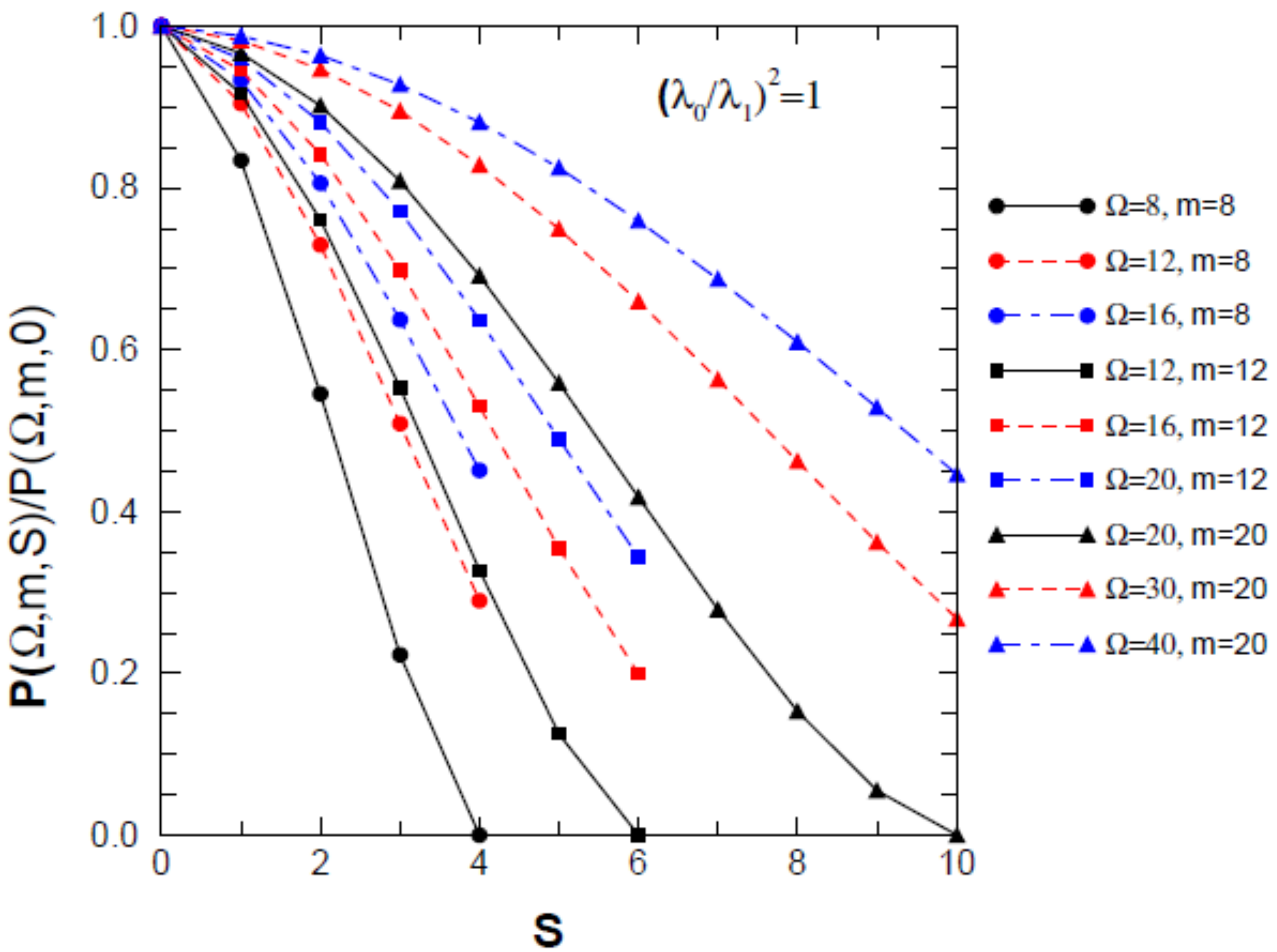
$$\begin{aligned}
&P(\Omega,m,S)=\frac{1}{\Omega(\Omega+1)/2}\left[\frac{\Omega+2}{\Omega+1}\mathcal{Q}^1(\{2\}:m,S)+\frac{\Omega^2+3\Omega+2}{\Omega^2+3\Omega}\mathcal{Q}^2(\{2\}:m,S)\right]\\
&+\frac{1}{\Omega(\Omega-1)/2}\left[\frac{\Omega+2}{\Omega+1}\mathcal{Q}^1(\{1^2\}:m,S)+\frac{\Omega^2+\Omega+2}{\Omega^2+\Omega}\mathcal{Q}^2(\{1^2\}:m,S)\right]\;;\\
&\mathcal{Q}^1(\{2\}:m,S)=\left[(\Omega+1)\mathcal{P}^0(m,S)/16\right]\left[m^x(m+2)/2+\mathcal{S}^2\right]\;,\\
&\mathcal{Q}^2(\{2\}:m,S)=\left[\Omega(\Omega+3)\mathcal{P}^0(m,S)/32\right]\left[m^x(m^x+1)-\mathcal{S}^2\right]\;,\\
&\mathcal{Q}^1(\{1^2\}:m,S)=\frac{(\Omega-1)}{16(\Omega-2)}\left[(\Omega+2)\,\mathcal{P}^1(m,S)\,\mathcal{P}^2(m,S)\right.\\
&\left.+\,8\Omega(m-1)(\Omega-2m+4)\mathcal{S}^2\right]\;,\\
&\mathcal{Q}^2(\{1^2\}:m,S)=\frac{\Omega}{8(\Omega-2)}\left[(3\Omega^2-7\Omega+6)(\mathcal{S}^2)^2\right.\\
&+3m(m-2)m^x(m^x-1)(\Omega+1)(\Omega+2)/4\\
&\left.+\,\mathcal{S}^2\left\{-mm^x(5\Omega-3)(\Omega+2)+\Omega(\Omega-1)(\Omega+1)(\Omega+6)\right\}\right]\;,\\
&\mathcal{P}^0(m,S)=\left[m(m+2)-4\mathcal{S}^2\right]\;,\quad \mathcal{P}^1(m,S)=\left[3m(m-2)+4\mathcal{S}^2\right]\;,\\
&\mathcal{P}^2(m,S)=3m^x(m-2)/2-\mathcal{S}^2\;,\quad m^x=\left(\Omega-\frac{m}{2}\right)\;,\quad \mathcal{S}^2=S(S+1)\;.
\end{aligned}$$

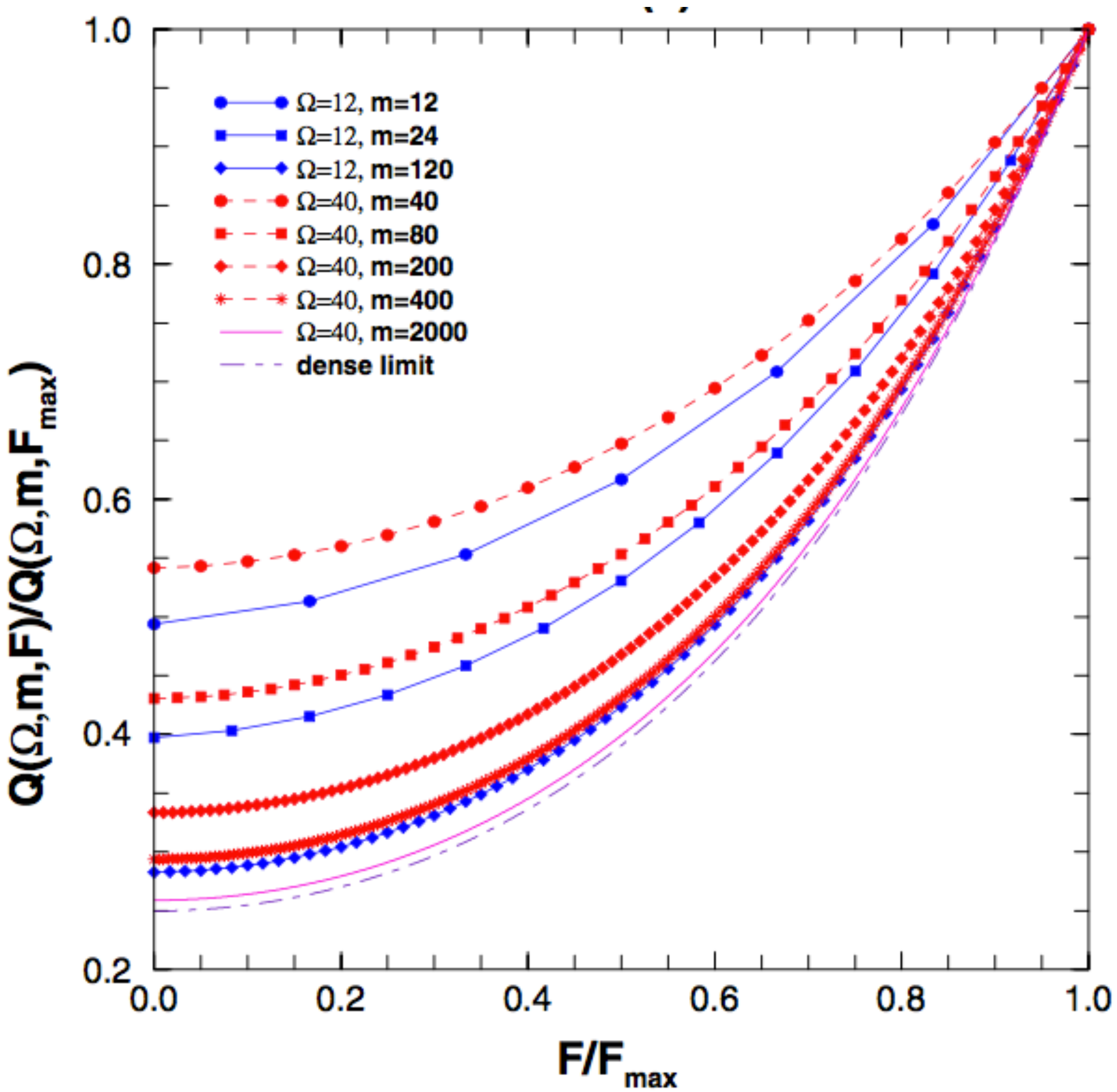
For fixed sp energies,

$$\sigma^2_{h(1)}(m,S)=\frac{(\Omega+2)m(\Omega-m/2)-2\Omega S(S+1)}{(\Omega-1)(\Omega+1)}\sigma^2_{h(1)}\left(1,\frac{1}{2}\right)$$

$$\overline{\sigma^2_{V(2)}(\Omega,m,S)} = \lambda^2 P(\Omega,m,S)$$

$$\overline{\sigma^2_{V(2)}(\Omega,m,F)} = \lambda^2\, Q(\Omega,m,F)$$





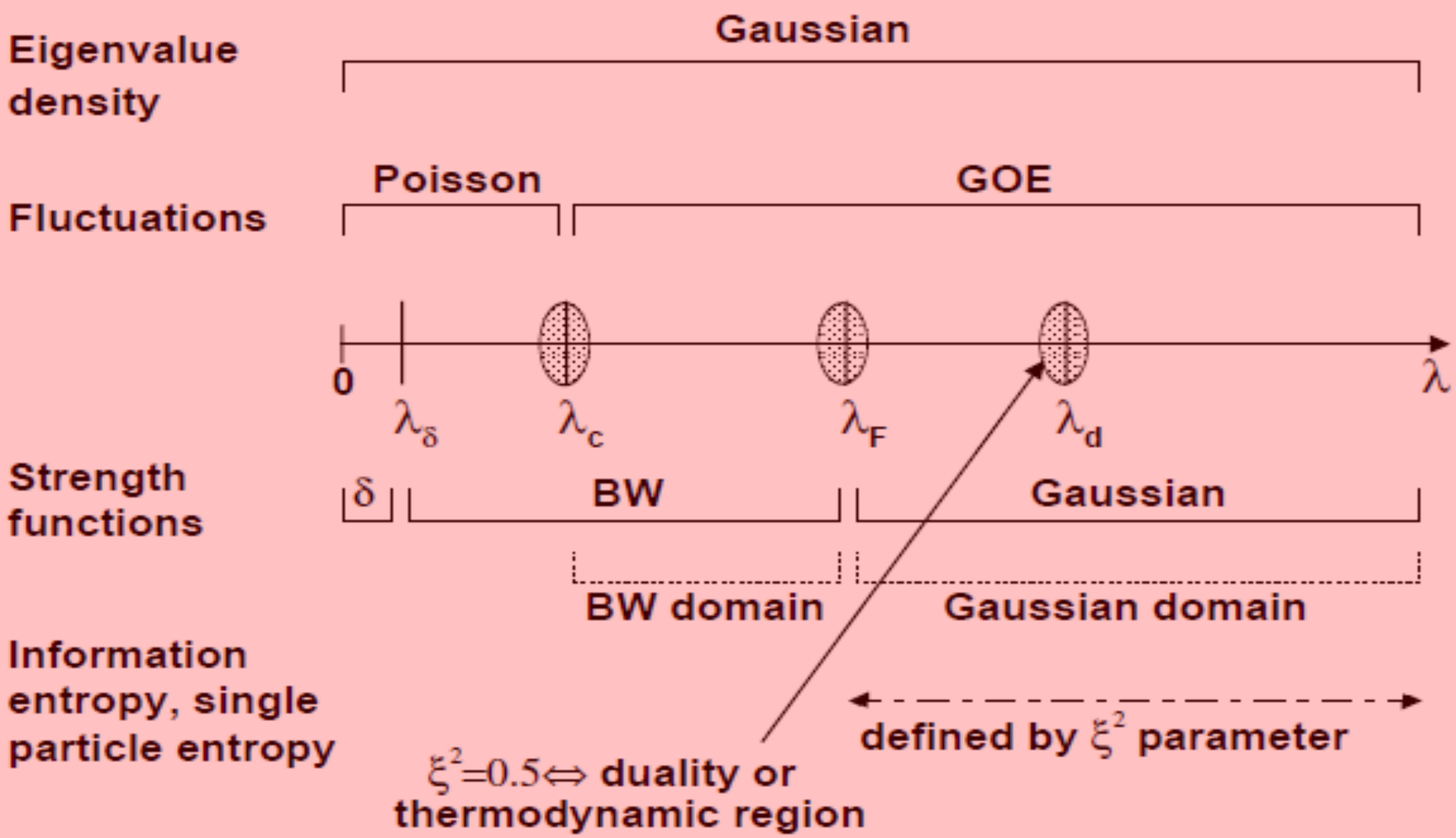
Propagator plays important role in determining the chaos markers generated by EE(1+2)-*s*

Chaos markers generated by EE(1+2)-S

Transition markers λ_c, λ_F and λ_d for EGOE(1+2)−s

$$H_\lambda = h(1) + \lambda\,[\,V^{s=0}(2) + V^{s=1}(2)\,]$$

parameters: (Ω, m, S, λ)



$$\lambda_c(S) \propto \frac{\Omega}{P(\Omega, m, S)}$$

$$\lambda_F(S) \propto \sqrt{\frac{m\Omega^2}{P(\Omega, m, S)}}$$

$$\lambda_d(S) \propto \sqrt{\frac{X(\Omega, m, S)}{P(\Omega, m, S)}}$$

The transition or chaos markers defined by the random interaction matrix ensembles with spin symmetry are important in understanding entanglement properties in multi-qubit systems and also allow us to define a region of thermalization in complex finite quantum systems modeled by these ensembles. Recent years have witnessed a renewed interest in qualitatively reassessing quantitatively and exploring many-body quantum complexity and quantum chaos implications in QIS.

V.K.B. Kota, A. Relano, J. Retamosa, and Manan Vyas, J.Stat. Mech. P10028 (2011); Manan Vyas, V.K.B. Kota and N.D. Chavda, Phys. Rev. E 81, 036212/1-17 (2010).
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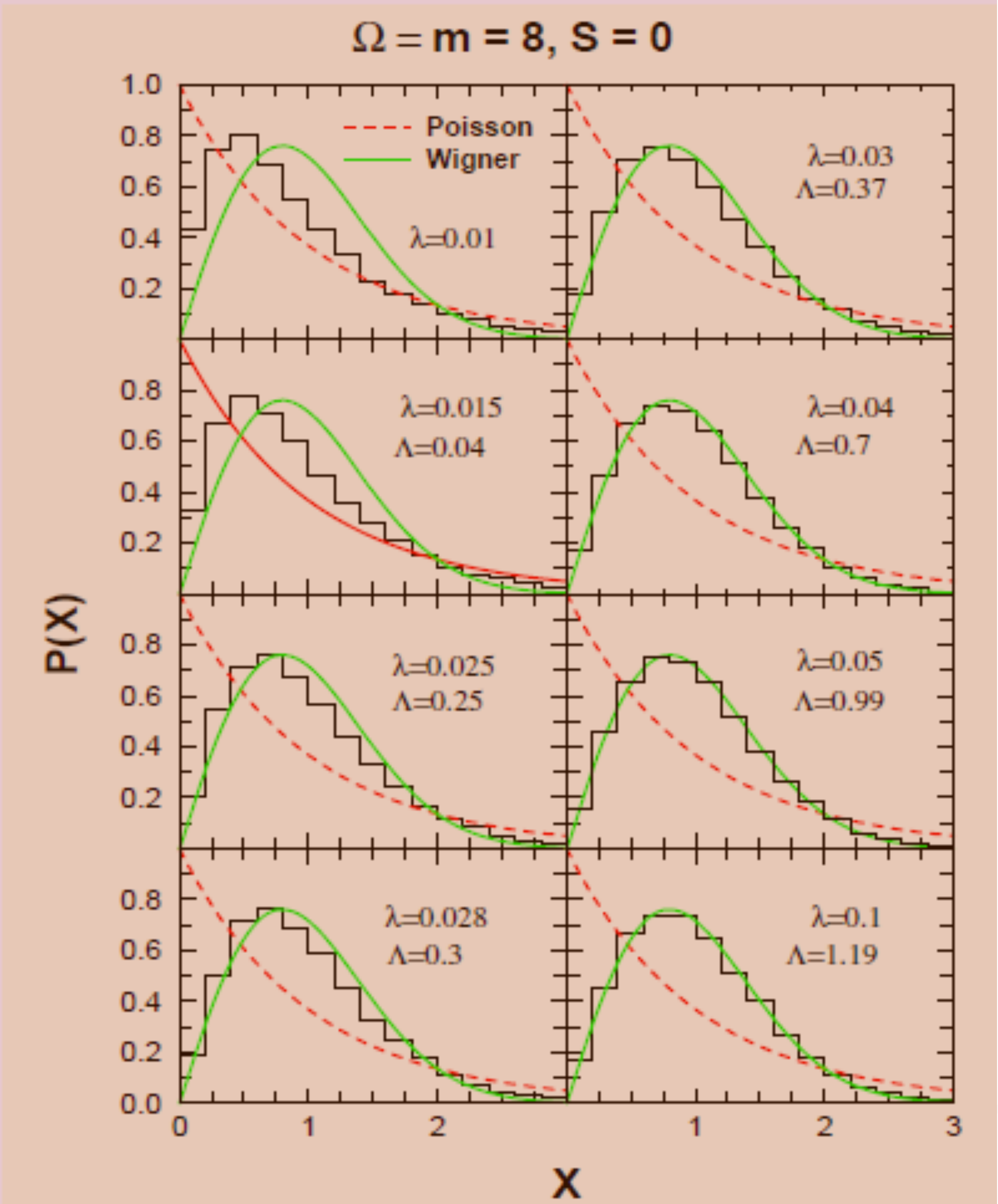
Poisson to GOE transition in NNSD



BW to Gaussian transition in Strength function

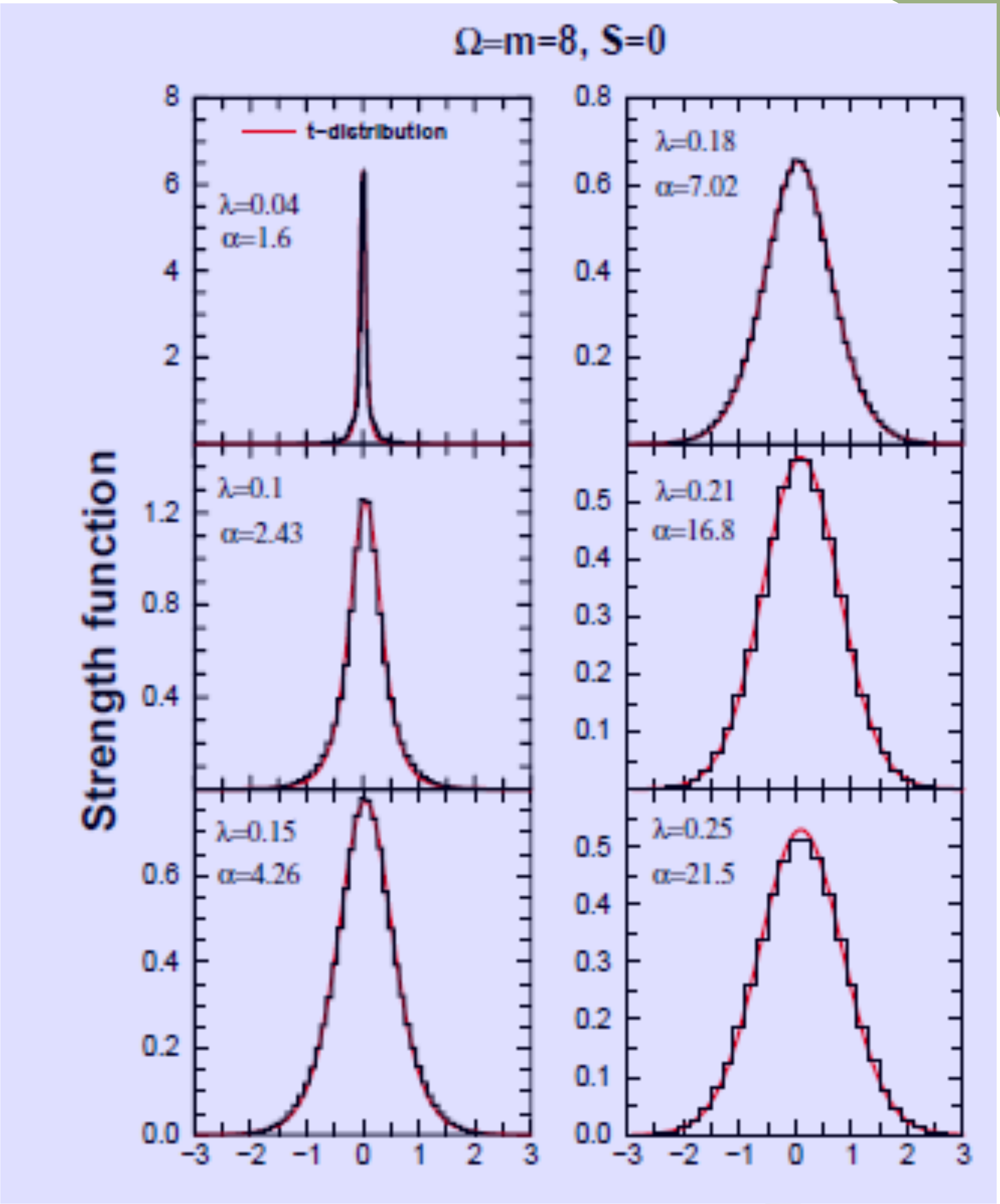
Expanding the mean-field $h(1)$ basis states in the eigenbasis

$$|k,S,M_S\rangle = \sum_E C_{k,S}^{E,S} |E,S,M_S\rangle,$$

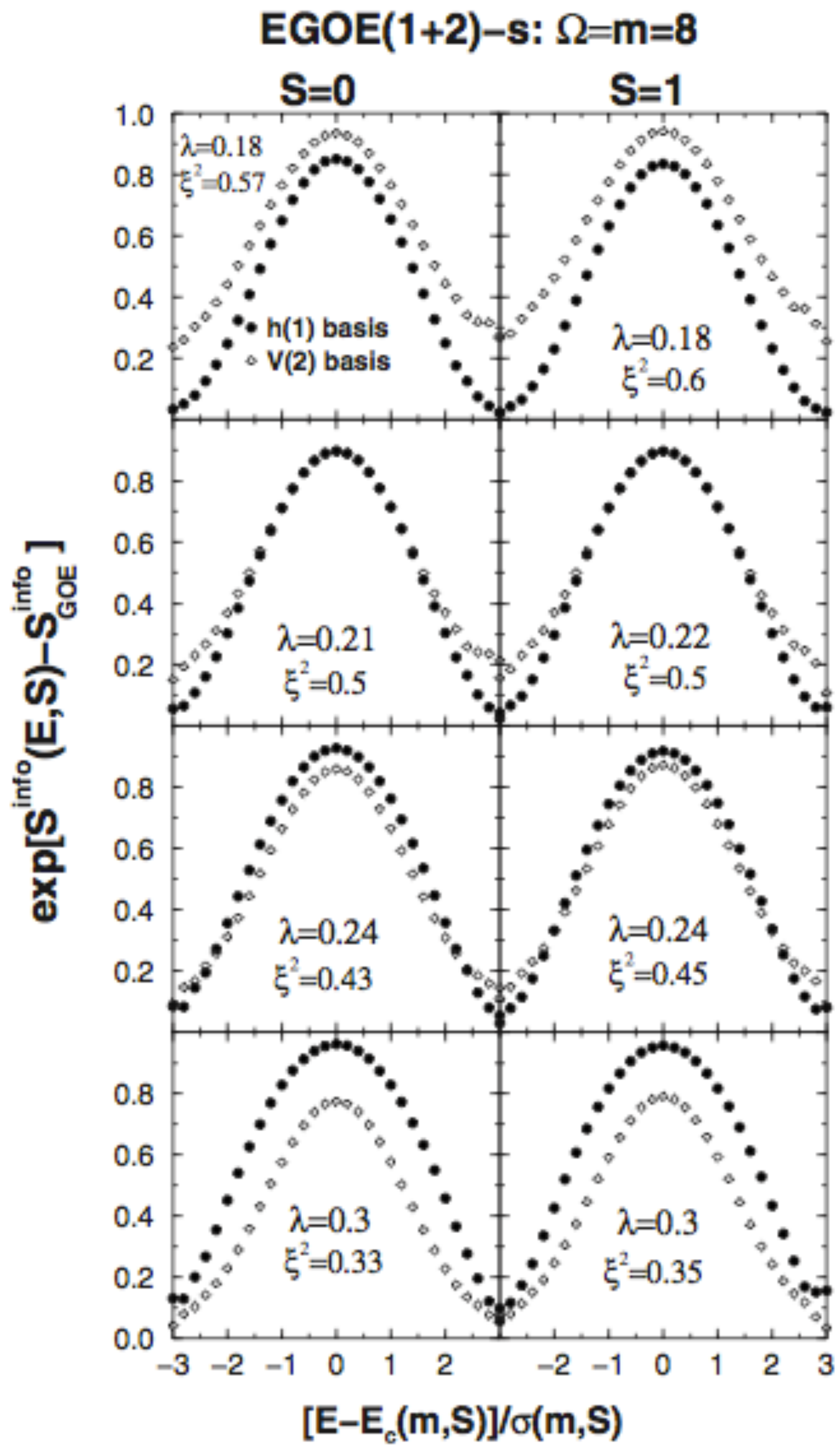
$$F_{k,S}(E,S) = |C_{k,S}^{E,S}|^2 d(m,S)\rho^{m,S}(E)$$

$$S^{\inf o}(E,S) = -\frac{1}{d(m,S)\rho^{m,S}(E)}$$

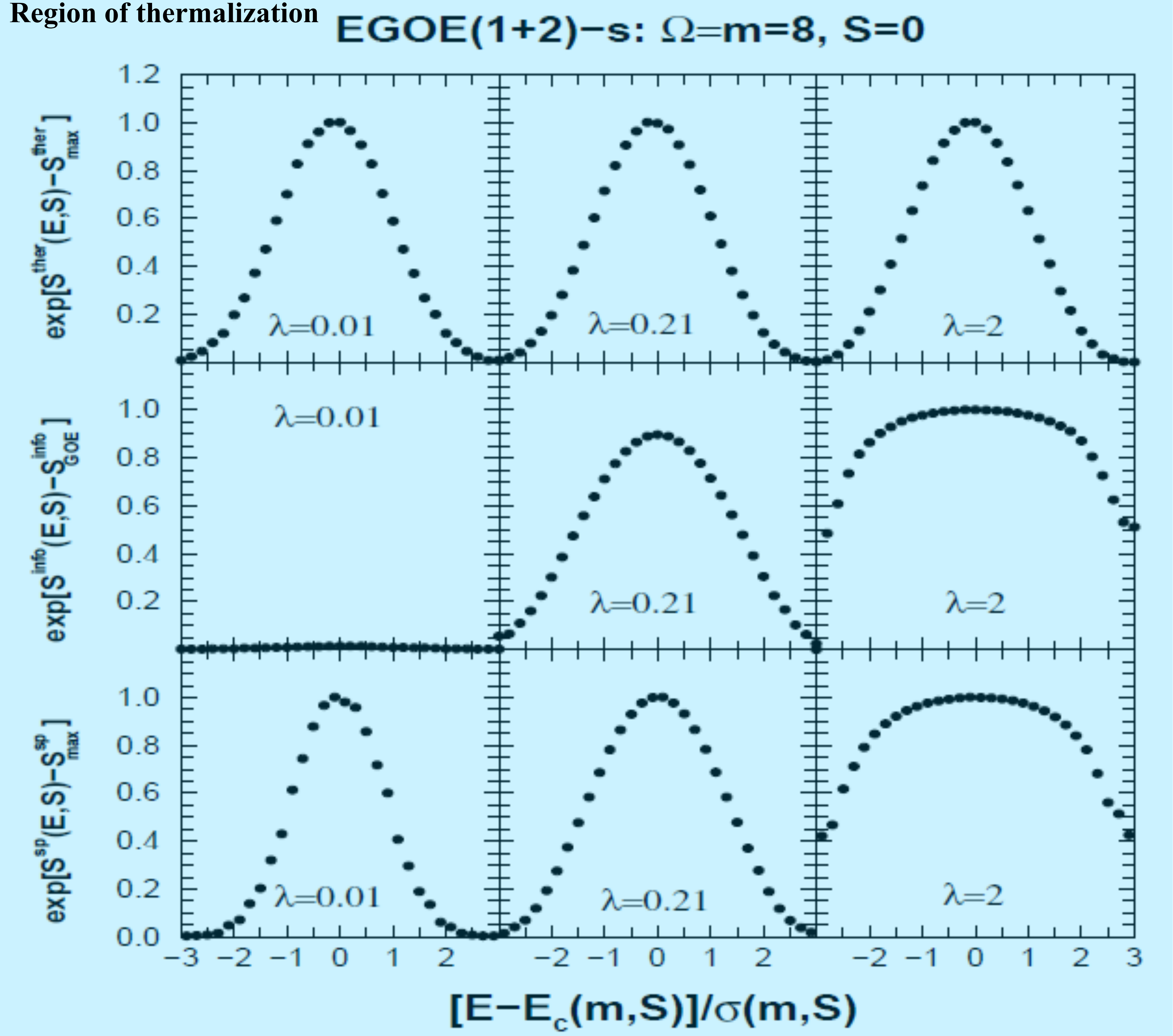
$$\times \sum_{E'} |C_{k,S}^{E',S}|^2 \ln |C_{k,S}^{E',S}|^2 \delta(E-E')$$



Information/Shannon entropy



Region of thermalization



Nuclear Shell model: M. Horoi, V. Zelevinsky, B.A. Brown, Phys. Rev. Lett. 74, 5194–5197 (1995)

Quench dynamics

$$H = h(1) + \lambda V(2)$$

$$\overset{E}{|\Psi\rangle} \neq |k\rangle$$

$$|\Psi(0)\rangle = |k\rangle$$

after time *t*,

$$\Psi(t) = |k(t)\rangle = \exp -iHt\,|k\rangle$$

$$= \sum_E C_k^E\, e^{-iEt}\,|E\rangle$$

$$C_k^E = \langle E \mid k\rangle$$

Transition probability

$$W_{k\to f}(t) = |\langle f \mid k(t)\rangle|^2 = |A_{k\to f}(t)|^2$$

$$A_{k\to f}(t) = \sum_E \left(C_f^E\right)^* C_k^E \exp -iEt$$

Fidelity

$$W_{k\to k}(t) = |A_{k\to k}(t)|^2 = \left|\sum_E \left[C_k^E\right]^2 \exp -iEt\right|^2$$

$$= \left|\int F_k^{m,S}(E) \exp -iEt \, dE\right|^2$$

survival probability, non-decay probability, return probability

Strength functions/LDOS $F_k^{m,S}(E) \;=\; |C_k^E|^2 \, \rho^{m,S}(E)$

for very short times,

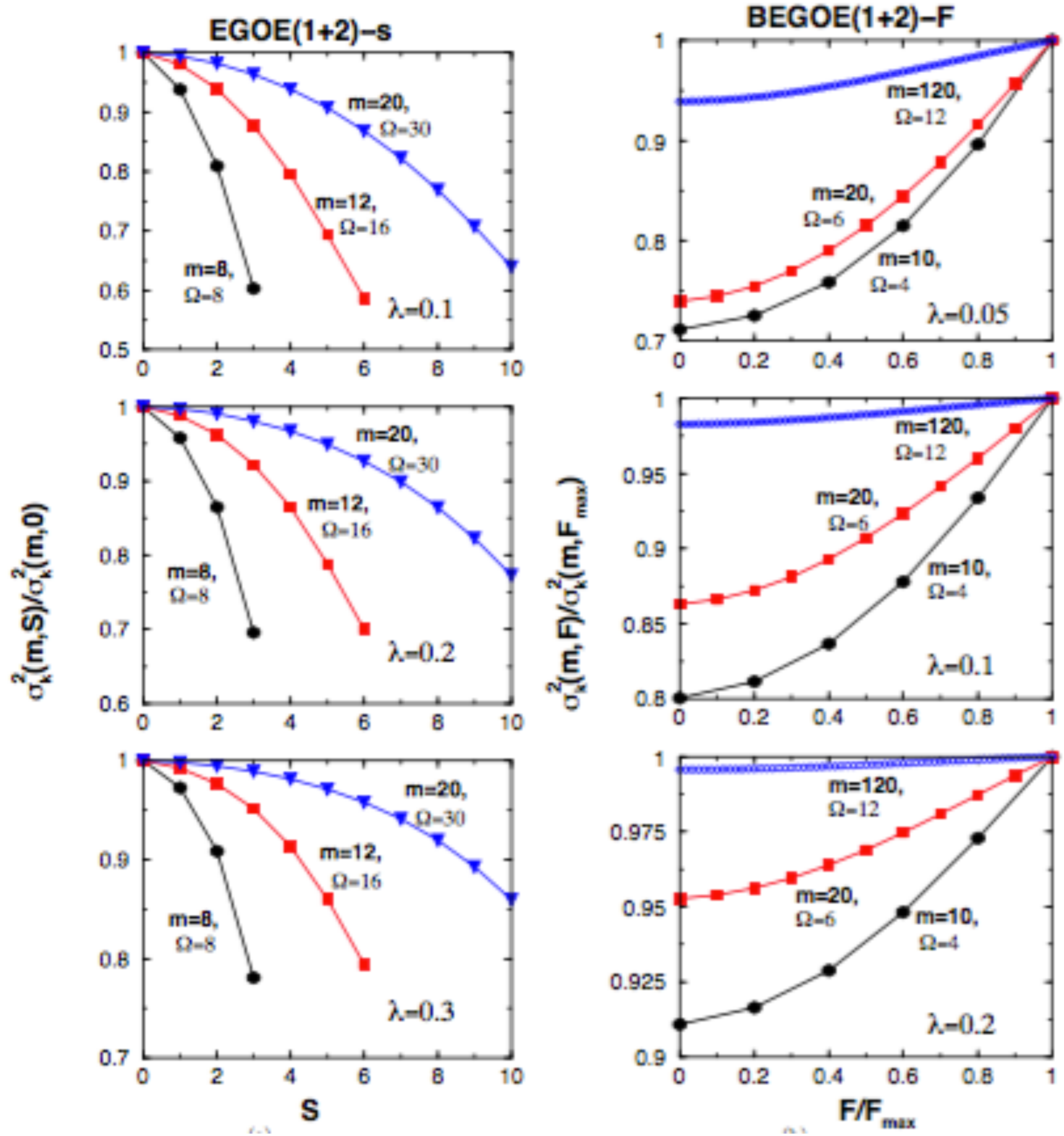
$$W_{k\to k}(t) = 1 - \sigma_k^2(m,S)\, t^2$$

$$e_k(m,S) \;=\; \langle k \mid H \mid k \rangle$$

$$\sigma_k^2(m,S) \;=\; \langle k \mid H^2 \mid k \rangle \;-\; e_k^2(m,S)$$

R. A. Jalabert and H. M. Pastawski, Phys. Rev. Lett. 86, 2490 (2001), T. Prosen and M. Žnidarič, J. Phys. A: Math. Gen. 35, 1455 (2002), T. Prosen, Phys. Rev. E 65, 036208 (2002), T. Gorin, T. Prosen, T. H. Seligman, and M. Žnidarič, Phys. Rep. 435, 33 (2006)

$$\overline{\sigma_k^2(m,S)} = \lambda^2 \, \frac{\overline{\sigma^2_{V(2)}(m,S)}}{\overline{\sigma^2_H(m,S)}}$$



$$W_{k\to k}(t) \xrightarrow{\text{BW region}} \exp -\Gamma(m,S)\,t$$

$$W_{k\to k}(t) \xrightarrow{\text{Gaussian region}} \exp -\sigma_k^2(m,S)\,t^2$$

BW to Gaussian transition region:

$$F_k^{m,S}(x:\nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi}\sqrt{\nu}\,\Gamma\left(\frac{\nu}{2}\right)}\left(\frac{x^2}{\nu}+1\right)^{-\dfrac{(\nu+1)}{2}} dx$$

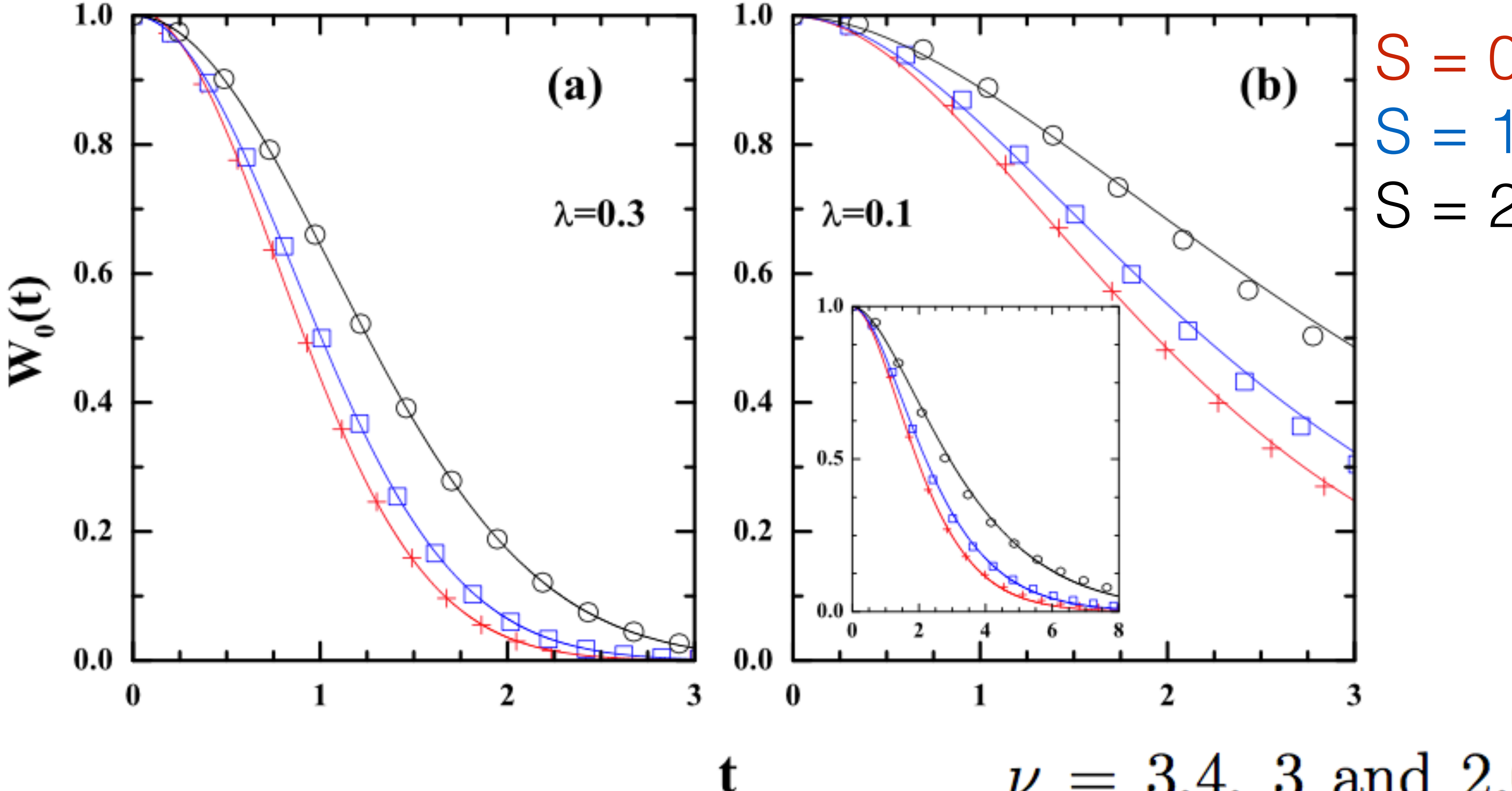
$$W_{k\to k}(t) \xrightarrow{\text{transition region}} \left|\frac{2^\nu\left(\sqrt{\nu}\right)^\nu}{\Gamma(\nu)}\int_0^\infty dx\,[x(x+|t'|)]^{(\nu-1)/2}\; e^{-\sqrt{\nu}\,(2x+|t'|)}\right|^2$$

$$t' = \sqrt{\frac{\beta(\nu+1)}{2\nu}}\,t\,. \qquad \alpha = (\nu+1)/2 \qquad \sigma_k^2(m,S) = \tfrac{\alpha}{2\alpha-3}\beta;\; \alpha > 3/2$$

(a) for $\nu = 1$, we get BW form for $F_k^{m,S}(E)$ with $\beta = \Gamma^2/4$ and (b) for $\nu \to \infty$, we get Gaussian form with $\sigma_k^2(m,S) = \beta/2$.

I. Dreier and S. Kotz, Statistics and Probability Letters 57, 221 (2002)

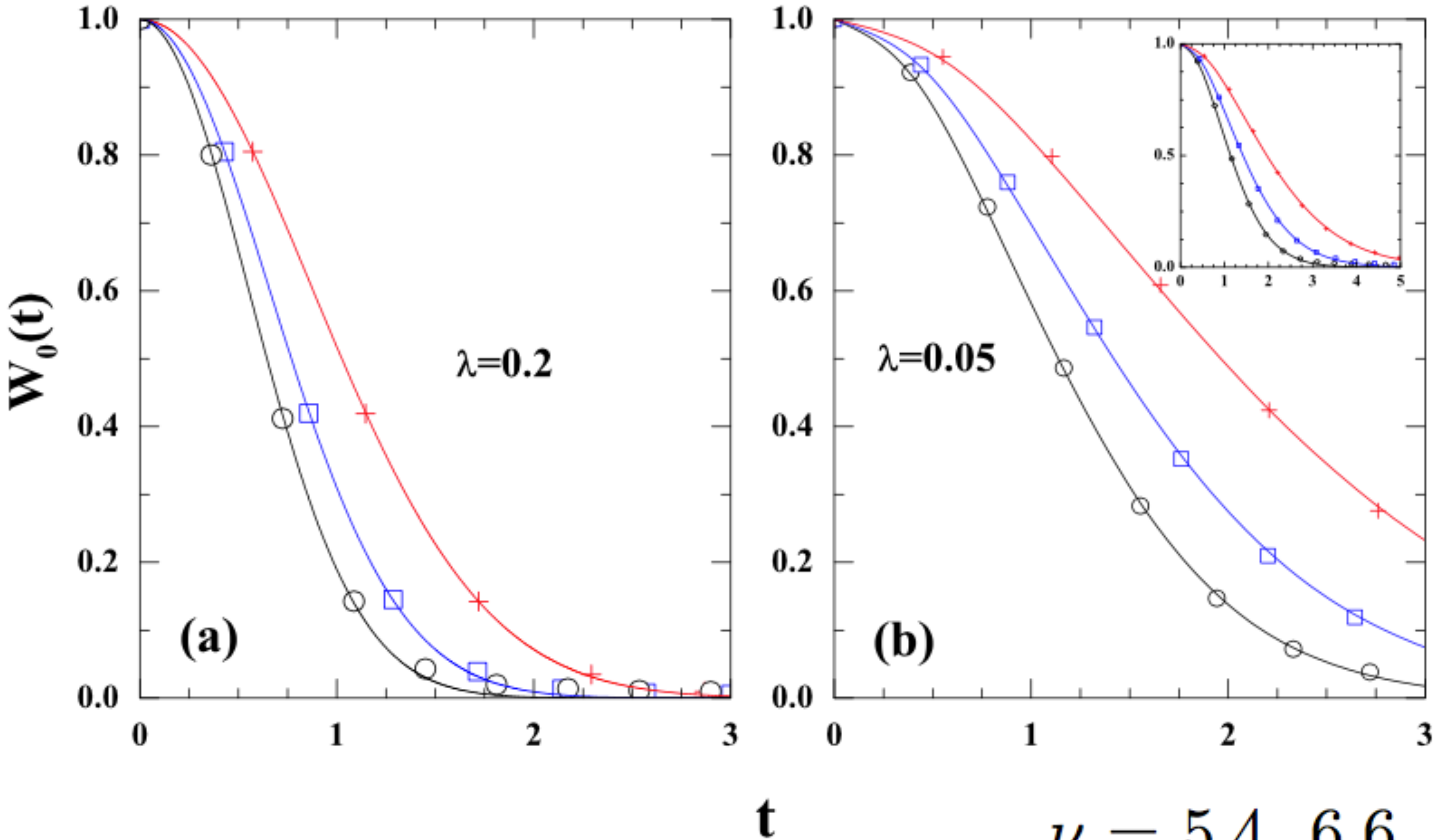
$\Omega = m = 8 \qquad d(m, S) = 1764,\ 2352 \text{ and } 720.$



$\nu = 3.4,\ 3 \text{ and } 2.6$

$\Omega = 4$
$m = 10$

$d(m, F) = 750,\ 594$ and 286 for $F = 2,\ 4$ and 5



F = 2
F = 4
F = 5

$\nu = 5.4,\ 6.6,$ and 10.0

Information entropy

Suppose there are *D+1* mean-field basis states such that

$f = 0, 1, \ldots, D$ with $|f = 0\rangle = |k\rangle$

$$S(t) = -\sum_{f=0}^{D} W_{k\to f}(t) \ln W_{k\to f}(t)$$

$$\sum_{f=0}^{D} W_{k\to f}(t) = 1$$

$$\begin{aligned} W_{k\to f}(t) &= \sum_{E} \left|C_0^E\right|^2 \left|C_f^E\right|^2 + 2 \sum_{E>E'} C_0^E C_f^E C_0^{E'} C_f^{E'} \cos(E-E')\, t \\ &= W_{k\to f}^{avg}(t) + W_{k\to f}^{flu}(t)\,. \end{aligned}$$

for very short times,

$$S(t) \longrightarrow \sigma_k^2(m,S)t^2 - t^2 \sum_{f=1}^{D} H_{0f}^2 \ln\left\{H_{0f}^2 t^2\right\}$$

$$H_{0f}^2 = |\langle f \mid H \mid 0\rangle|^2$$

E. J. Torres-Herrera, Manan Vyas and L. F. Santos, New Journal of Physics 16, 063010/1-33 (2014), L.F. Santos, F. Borgonovi, and F. M. Izrailev, Phys. Rev. Lett. 108, 094102 (2012), V. V. Flambaum and F. M. Izrailev, Phys. Rev. E 64036220 (2001)

Assume that there are n number of mean-field basis states f that contribute in the sum.

$$\lambda > \lambda_c \rightarrow W_{k\to f} \text{ by its average value } \overline{W}.$$

$$S(t) = -W_0(t) \ln W_0(t) - \sum_{r=1}^{n} \overline{W} \ln \overline{W} \; ; \quad \overline{W} = \frac{1 - W_0}{n}$$

$$= -W_0(t) \ln W_0(t) - [1 - W_0(t)] \ln \left(\frac{1 - W_0(t)}{n} \right) .$$

$$W_{k\to k}(t) = W_0(t)$$

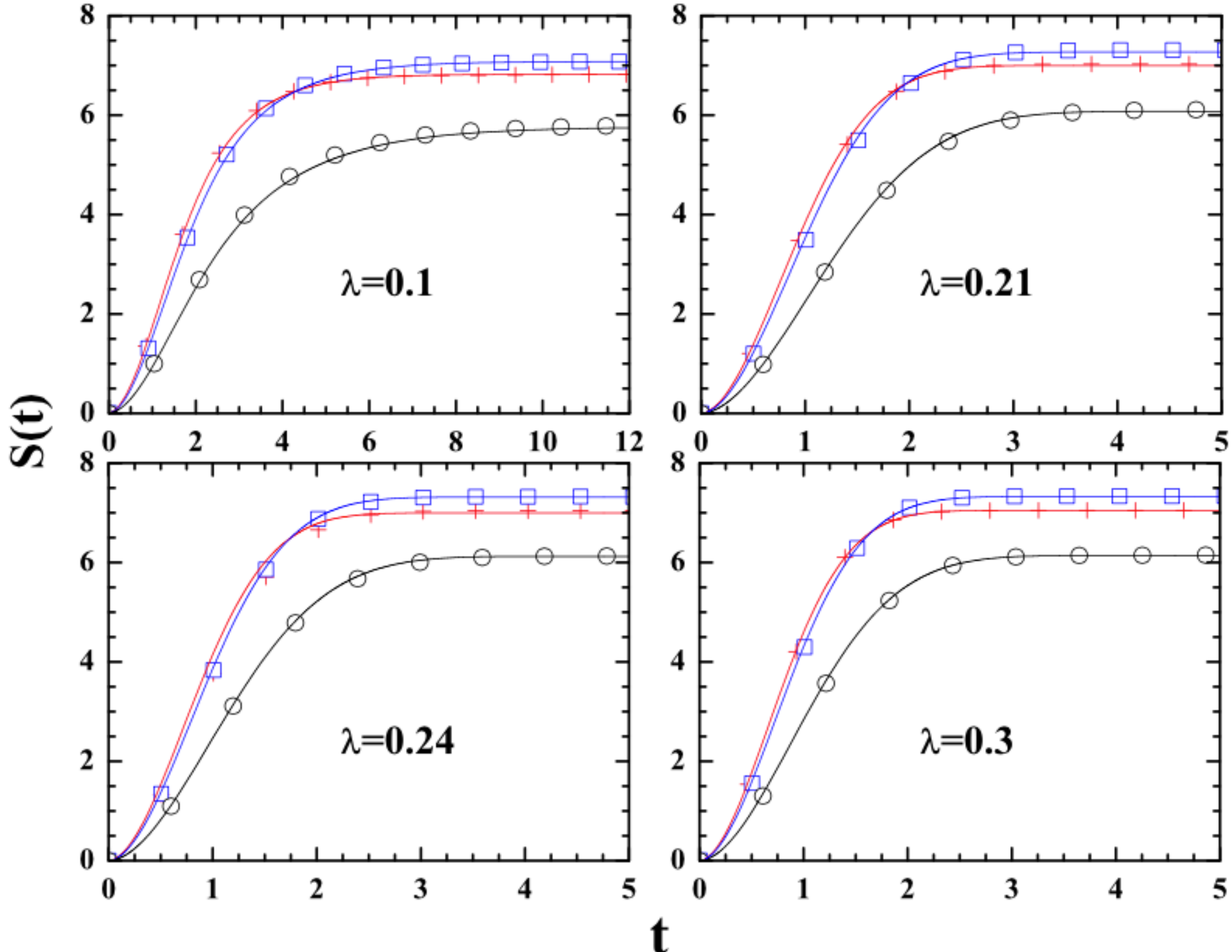
$$\textstyle\sum_{f\neq 0} 1 = n$$

$$n = e^{S(\infty)}$$

$$n \sim \kappa \text{ NPC}$$

$\Omega = m = 8 \qquad d(m, S) = 1764,\ 2352 \text{ and } 720.$

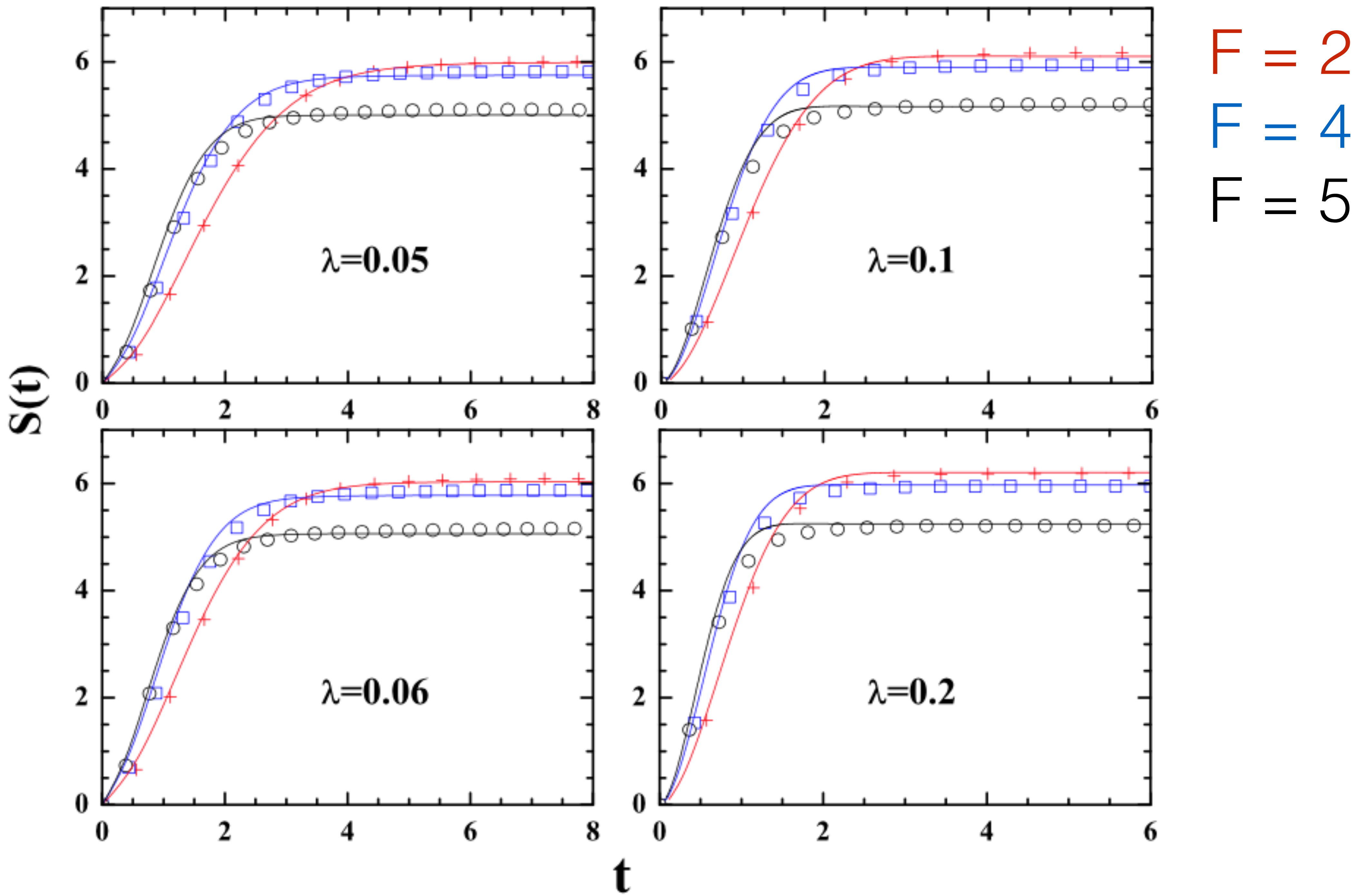
S = 0
S = 1
S = 2



$\kappa = 2$ in the Gaussian region and $\kappa = 3$, 3.3 and 3.6 for $S = 0-2$ respectively, in the BW to Gaussian transition region.

$\Omega = 4$
$m = 10$

$d(m, F) = 750,\ 594$ and 286 for $F = 2,\ 4$ and 5



$\kappa = 2$ in the Gaussian region and $\kappa = 2.8$, 2.55 and 2.3 for $F = 2$, 4 and 5 in the BW to Gaussian transition region.

Saturation time:

EGOE(1+2)-S				BEGOE(1+2)-F			
	t_{sat}				t_{sat}		
λ	$S=0$	$S=1$	$S=2$	λ	$F=2$	$F=4$	$F=5$
0.1	7.97	9.62	11.49	0.05	6.74	4.62	3.28
0.21	3.32	3.73	4.21	0.06	5.57	3.87	3.15
0.24	3.13	3.5	3.97	0.1	3.66	2.68	2.2
0.3	2.7	3.2	3.65	0.2	3.07	2.1	1.7

Conclusions

An isolated interacting many-body fermionic(bosonic) quantum system exhibits delay in relaxation with increasing(decreasing) many-particle spin.

Our results may contribute to formulation of a general statistical description of non-equilibrium many-body quantum systems.