Cooperative shielding in many-body systems with long-range interaction

F. Borgonovi, Luca G. Celardo (BUAP, Puebla, Mexico) , Lea F. Santos (Yeshiva Univ., New York, USA), R. Bachelard (IFSC-USP, Sao Carlo, Brasil), R. Kaiser (INLN, Nice France)

Istituto Nazionale di Fisica Nucleare (Pavia, ITALY)
Interdisciplinary Laboratories for Advanced Materials Physics
Dept of Math. and Phys., Catholic Univ., (Brescia, ITALY)

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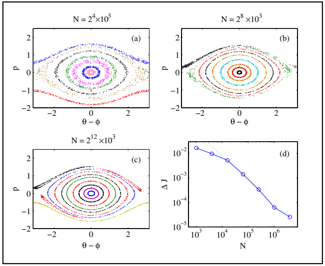
Relevant publications

- (many body) Lea F. Santos, Fausto Borgonovi and Giuseppe Luca Celardo, Phys. Rev. Lett. 116, 250402 (2016)
- (transport) G. L. Celardo , R. Kaiser, F. Borgonovi, Phys. Rev. B 94, 144206 (2016);
- (classical vs quantum) in preparation with R. Bachelard, L.F. Santos, G.L. Celardo

Long Range Interactions

- Statistical and Dynamical properties (A. Campa, T. Dauxois, D. Fanelli, S. Ruffo, "Physics of Long-Range Interacting Systems" , Oxford University Press, (2014)
- Non-Extensivity, Non-Additivity
- Ensemble inequivalence "Inequivalence of Ensembles in a System with Long-Range Interactions" Julien Barré, David Mukamel, and Stefano Ruffo Phys. Rev. Lett. 87, 030601 (2001)
- Broken Ergodicity (F.Borgonovi, G.L.Celardo, M.Maianti and E.Pedersoli "Broken Ergodicity in Classically Chaotic Spin Systems" Jour. of Stat. Phys. 116 , (2004) 235
- long range $V_{i,j} = \frac{J}{r_{ij}^{\alpha}} \quad \alpha < d$
- Abundance of Regular orbits and suppression of chaos R. Bachelard, C. Chandre, D. Fanelli, X. Leoncini, and S. Ruffo Phys. Rev. Lett. 101, 260603 (2008)



Motivations

- Out-of-equilibrium dynamics in many-body systems.
- Surge of interest in long range: cold atomic clouds, light harvesting complexes, exciton wires, ion traps, etc..
- Cooperativity and Long Range interaction: Emergent quantum properties, Macroscopic quantum tunnelling, Cooperative propagation of information.
- Long-range interacting systems: broken ergodicity, Long Relaxation Times, long-lasting out-of-equilibrium regimes, Abundance of Regular Orbits

Many recent results

- Spreading of perturbations, correlations, entanglement, etc.. in many body systems.
- Lieb-Robinson bounds: for nearest neighbors interaction, spreading within a sound-cone with exponential suppression outside.
- Fast propagation of perturbations, Experiments with long range, P. Richerme et al., Nature 511 (2014) 198, P. Jurcevic et al, Nature, 511 (2014) 202
- Breakdown of Quasilocality in Long-Range Quantum Lattice Models Theory, J.Eisert, M van den Worm, S.R.Manmana M.Kastner, PRL 111, 260401 (2013); J. Schachenmayer et al, PRX 3, 031015 (2013); K. R. A. Hazzard, S. R. Manmana, M. Foss-Feig, and A. M. Rey, PRL 110, 075301 (2013), P. Hauke, L. Tagliacozzo PRL 111, 207202 (2013)
- but also "suppression" of long-range effects D.-M. Storch , M. van den Worm and M. Kastner, NJP 17 (2015) 063021

Out-of-equilibrium dynamics in MBS: Ion Traps

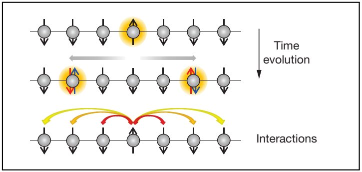
Many Body Hamiltonian:

$$H = B \sum_k \sigma_k^z + J \sum_{i<j} \frac{\sigma_i^x \sigma_j^x}{|i-j|^\alpha}$$

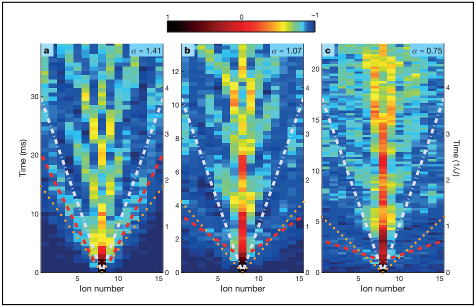
Experimental observation of spread of information and violation of Lieb-Robinson sound cone.

Ion Traps

P. Jurcevic et al, Nature, 511 (2014) 202 .



Sound Cones: Lieb-Robinson bounds



Red lines, fits to the observed magnon arrival times; white lines, sound cone for averaged nearest-neighbour interactions; orange dots, after renormalization by the algebraic tail.

Non local propagation of correlations

Slow growth of bipartite entanglement

PHYSICAL REVIEW X **3**, 031015 (2013)

Entanglement Growth in Quench Dynamics with Variable Range Interactions

J. Schachenmayer,[1] B. P. Lanyon,[2] C. F. Roos,[2] and A. J. Daley[1]

[1]*Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, Pennsylvania 15260, USA*
[2]*Institute for Quantum Optics and Quantum Information of the Austrian Academy of Sciences and Institute for Experimental Physics, University of Innsbruck, Innsbruck, Austria*

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Studying entanglement growth in quantum dynamics provides both insight into the underlying microscopic processes and information about the complexity of the quantum states, which is related to the efficiency of simulations on classical computers. Recently, experiments with trapped ions, polar molecules, and Rydberg excitations have provided new opportunities to observe dynamics with long-range interactions. We explore nonequilibrium coherent dynamics after a quantum quench in such systems, identifying qualitatively different behavior as the exponent of algebraically decaying spin-spin interactions in a transverse Ising chain is varied. Computing the buildup of bipartite entanglement as well as mutual information between distant spins, we identify linear growth of entanglement entropy corresponding to propagation of quasiparticles for shorter-range interactions, with the maximum rate of growth occurring when the Hamiltonian parameters match those for the quantum phase transition. Counterintuitively, the growth of bipartite entanglement for long-range interactions is only logarithmic for most regimes, i.e., substantially slower than for shorter-range interactions. Experiments with trapped ions allow for the realization of this system with a tunable interaction range, and we show that the different phenomena are robust for finite system sizes and in the presence of noise. These results can act as a direct guide for the generation of large-scale entanglement in such experiments, towards a regime where the entanglement growth can render existing classical simulations inefficient.

Sound-Cone features for long-range

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PAPER

Interplay of soundcone and supersonic propagation in lattice models with power law interactions

David-Maximilian Storch[1], Mauritz van den Worm[2,3] and Michael Kastner[2,3]

[1] Department of Physics, Georg-August-Universität Göttingen, Friedrich-Hund-Platz 1, D-7077 Göttingen, Germany
[2] National Institute for Theoretical Physics (NITheP), Stellenbosch 7600, South Africa
[3] Institute of Theoretical Physics, University of Stellenbosch, Stellenbosch 7600, South Africa

E-mail: kastner@sun.ac.za

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Abstract

We study the spreading of correlations and other physical quantities in quantum lattice models with interactions or hopping decaying like $r^{-\alpha}$ with the distance r. Our focus is on exponents α between 0 and 6, where the interplay of long- and short-range features gives rise to a complex phenomenology and interesting physical effects, and which is also the relevant range for experimental realizations with cold atoms, ions, or molecules. We present analytical and numerical results, providing a comprehensive picture of spatio-temporal propagation. Lieb–Robinson-type bounds are extended to strongly long-range interactions where α is smaller than the lattice dimension, and we report particularly sharp bounds that are capable of reproducing regimes with soundcone as well as supersonic dynamics. Complementary lower bounds prove that faster-than-soundcone propagation occurs for $\alpha < 2$ in any spatial dimension, although cone-like features are shown to also occur in that regime. Our results provide guidance for optimizing experimental efforts to harness long-range interactions in a variety of quantum information and signaling tasks.

Cooperative Shielding

We will show that such apparently contradictory behavior is caused by a general property of long-range interacting systems (Cooperative Shielding). It refers to shielded subspaces inside of which the evolution is unaffected by long-range interactions for a long time. As a result, the dynamics strongly depends on the initial state: if it belongs to a shielded subspace, the spreading of perturbation satisfies the Lieb-Robinson bound and may even be suppressed, while for initial states with components in different subspaces, the propagation may be quasi-instantaneous.

The Shielding effect : a trivial example

- Let us consider a system:

$$H = H_0 + V, \qquad \text{with} \qquad [H_0, V] = 0$$

with V highly degenerate $V|v_k\rangle = v|v_k\rangle$, $k = 1, .., g$.

- $|\psi_0\rangle = \sum_{k=1}^{g} c_k |v_k\rangle$

V contribute to the dynamics only with a global phase

$$|\psi(t)\rangle = e^{-iHt/\hbar}|\psi_0\rangle = e^{-ivt/\hbar}e^{-iH_0t/\hbar}|\psi_0\rangle$$

We have shielding from V!!

- H_0 describes completely the dynamics (within a constant phase) and we may call it emergent Hamiltonian

Many open questions for non trivial cases:

- What if $[H_0, V] \neq 0$?
- What if the spectrum of V is not degenerate?
- What's the connection with long-range interacting systems ?
- Is this a cooperative effects ? (how it scales with the number of particles?)
- What is and how to find, if any, the emergent Hamiltonian?

Cooperative Shielding in many-body systems

Experimentally accessible spin 1/2 1–d Hamiltonian:

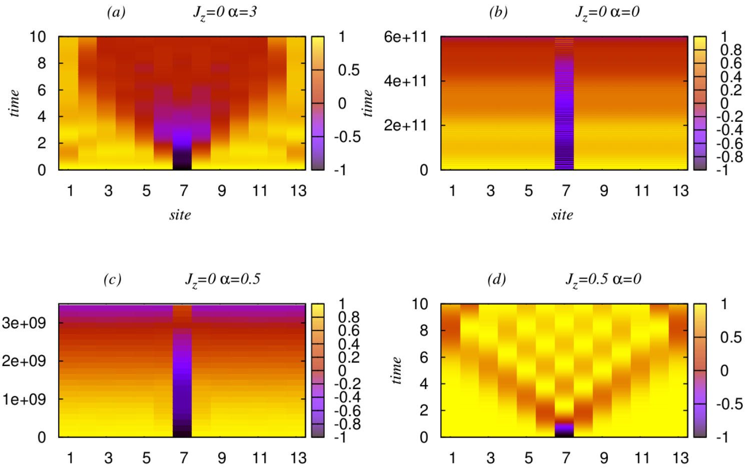
$$H = H_0 + V,$$
$$H_0 = B\sum_{n=1}^{L}\sigma_n^z + J_z\sum_{n=1}^{L-1}\sigma_n^z\sigma_{n+1}^z, \quad \text{neareast} - \text{neighbor}$$
$$V = J\sum_{n<m}\frac{1}{|n-m|^{\alpha}}\sigma_n^x\sigma_m^x. \quad \text{variable} \quad \text{range} - \alpha$$

- $\alpha < 1$: long range. $\alpha > 1$: short range. For $\alpha = 0$:

$$V = J\sum_{n<m}\sigma_n^x\sigma_m^x = \frac{JM_x^2}{2} - \frac{JL}{2} \quad \text{where} \quad M_x = \sum_n \sigma_n^x$$

has a degenerate spectrum, and its eigenvalues are given by,
$V_b = J(L/2-b)^2/2 - JL/2, \quad$ where $\quad b = 0, 1, \dots L/2$

Charge Propagation, $B = 1/2$



Initially : All spins up along x, but the central spin (-x);
a) short range $\alpha = 3 \rightarrow$ sound-cone;

d) infinite range $\alpha = 0$ & nearest neighbor $\rightarrow$, sound cone (effective short range)

In (a) where the interaction is short range $H_0 = B \sum_n \sigma_n^z$ effectively couples states belonging to different subspaces of V and this leads to the evident sound cone. In (b) and (c) (long range) the dynamics is frozen for a long time, which increases with the range of the interaction [compare the time scales in (b) and (c)]. The long-time localization of spin excitations is caused by the separated energy bands of V and the absence of direct coupling within the band. Notice that the energy bands for case (c) are no longer degenerate, yet localization persists for a long time. Since the initial state is not an eigenstate of the total Hamiltonian, the spin excitation does eventually spread and the spins reverse their signs. While for $\alpha < 1$ in the presence of an external field the dynamics is frozen, the addition of NN interaction restores the propagation of perturbations (d). Despite the existence of long-range interactions, the evolution can be described by an effective short-ranged Hamiltonian. This is the hallmark of the cooperative shielding effect.

Probability to stay within one band

In each band there is a g-degeneracy $k = 1, .., g$.

$$V|V_{bk}\rangle = V_b|V_{bk}\rangle$$

Taking a random superposition of degenerate eigenstates inside one band b one may ask for the probability to stay within the band b or to the leaking probability to go outside that band due to H_0,

$$|\psi(0)\rangle = \sum_k c_k|V_{bk}\rangle \to |\psi(t)\rangle$$

$$P_b(t) = \sum_k |\langle V_{bk}|\psi(t)\rangle|^2 \quad P_{leak} = \lim_{t\to\infty} 1 - P_b(t)$$

Leaking Probability

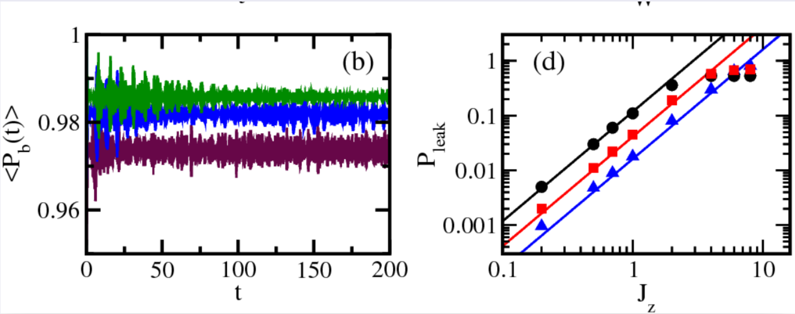
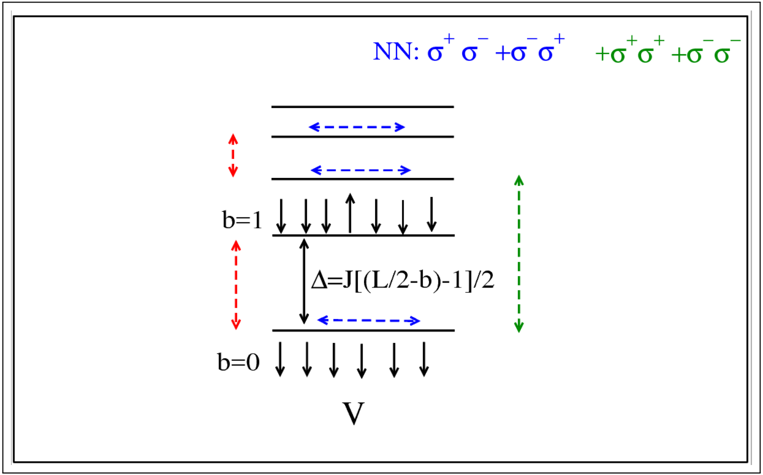


Figure: Left : average probability to stay within the band $b = 1$. Right: Leaking probability to go out of that band. Here is $J_z = 1/4, J = 1, L = 10; 12; 14$ from bottom to top. Perturbative estimate : $P_{leak} \propto (J_z/J)^2/L$.

Effective couplings ($\alpha = 0$)



Analogy with Quantum Zeno Effect

Consider the total Hamiltonian $H = H_0 + KH_{meas}$, which one may interpret as a quantum system described by H_0 that is continuously observed by an "apparatus" characterized by KH_{meas} . In the limit of strong coupling, $K \to \infty$, a superselection rule is induced that splits the Hilbert space into the eigensubspaces of KH_{meas}. Each one of these invariant quantum Zeno subspaces is specified by an eigenvalue and is formed by the corresponding set of degenerate eigenstates of KH_{meas} . The dynamics becomes confined to these P subspaces and dictated by the Zeno Hamiltonian

P. Facchi and S. Pascazio: Phys. Rev. Lett. 89 (2002)

$$H_z = \sum_b P_b H_0 P_b + V_b P_b = diag(H_0) + \sum_b V_b P_b$$

where P_b are the projectors on the eigensubspace of V corresponding to V_b.

The emergent Hamiltonian

Consider the total Hamiltonian $H = H_0 + V_{LR}$ can be written in the basis of $V_{LR} = \sum_b V_b \sum_j |V_{bj}\rangle\langle V_{bj}|$,

$$H = \sum_{b,b'} \sum_{j,k} |V_{bj}\rangle\langle V_{bj}|H_0|V_{b'k}\rangle\langle V_{b'k}| + \sum_b V_b \sum_j |V_{bj}\rangle\langle V_{bj}|$$

taking the mean field approximation one gets

The Zeno Hamiltonian

$$H_z = \sum_{bj} (V_b + \langle V_{bj}|H_0|V_{bj}\rangle) |V_{bj}\rangle\langle V_{bj}|$$

Dynamics inside one specific band b

- Specifically we should consider the diagonal matrix elements of H_0 : $\langle V_{bj}|H_0|V_{bj}\rangle$
- H_0 represents a nearest neighbor (NN) coupling with strength $J_z \neq 0$
- the Zeno hamiltonian reads,

$$H_z = V_b \sum_j |V_{bj}\rangle\langle V_{bj}| + \frac{J_z}{4}\sum_{n=1}^{L-1}\left(\sigma_n^+\sigma_{n+1}^- + \sigma_n^-\sigma_{n+1}^+\right)$$

and one has an **effective NN interaction** which conserves the number of excitations inside each band b.

Zeno Fidelity

To substantiate that the dynamics becomes indeed controlled by the Zeno Hamiltonian as L increases, we consider an initial random state inside the b-th band of V

$$|\Psi(0)\rangle = \sum_k c_k |V_{bk}\rangle$$

and the overlap of its evolution under both the total Hamiltonian H and H_z (fidelity)

$$F(t) = |\langle\Psi(0)|e^{iH_z t/\hbar} e^{-iHt/\hbar}|\Psi(0)\rangle|^2.$$

It is clear that if in some limit $H \to H_z$ then $F(t) \to 1$.

Numerical Experiments

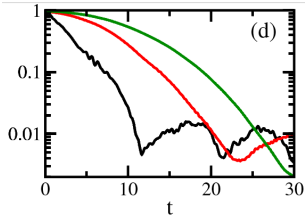


Figure: Decay of fidelity in time : black (L=10), red (L=12), green (L=14)

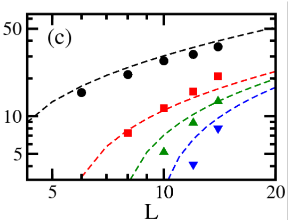


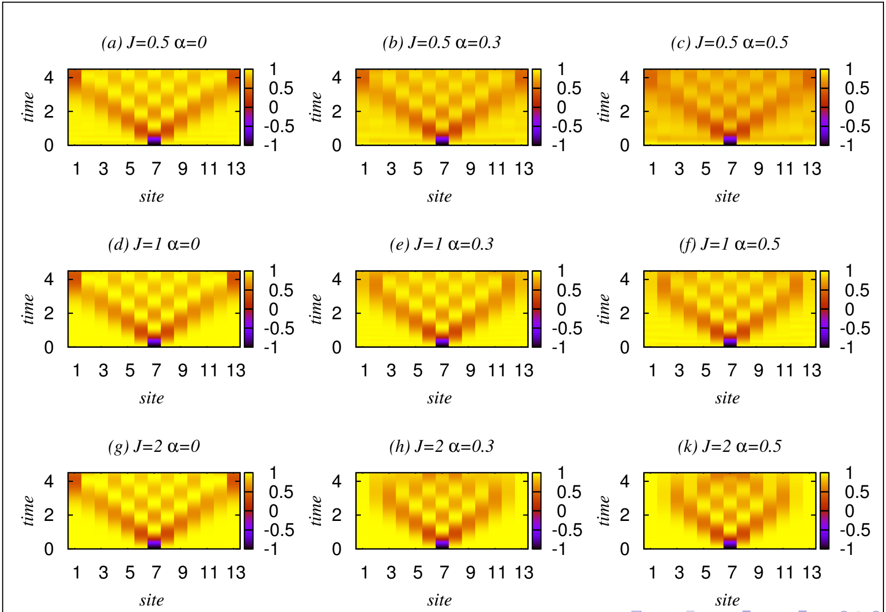
Figure: Shielding time $T_{1/2}$ *vs* L, black ($b = 1$), red ($b = 2$), green ($b = 3$). Dashed = perturbation theory.

Spreading of perturbation

Starting with an initial state inside one specific band the dynamics is dictated by the Zeno Hamiltonian up to $T_{1/2}$. In other words we will have a spreading of perturbation effectively short-ranged despite the presence of long range interaction.

One might think that is is an effect due to the presence of gaps between the degenerate bands occurring for $\alpha = 0$. Nevertheless this is not a peculiarity of $\alpha = 0$. Indeed even for $\alpha \neq 0$ when eigenvalues are not degenerate anymore shielding is numerically found!

Shielding : independence of J coupling



Conclusions and Perspectives

1. Shielding is a novel cooperative effect : since it preserves invariant coherent subspaces, and its robustness increases with the system size, it might be essential in building efficient quantum devices able to work at room temperature.
2. Shielding allows to control quantum dynamics since the spreading of correlations strongly depends on the initial state.
3. Transport properties are strongly affected from shielding (Phys. Rev. B 94, 144206 (2016))
4. Is it possible to have Classical Shielding? in preparation with L. Celardo, L. F. Santos & R. Bachelard

Classical Shielding : a renormalized model

Let us consider the Hamiltonian ($\alpha = 0$, $J > 0, J_x > 0$) with a renormalization factor,

$$H_{LR} + H_{SR} = -\frac{J}{2N^*} \sum_{k=1}^{N} \sum_{j \neq k} S_k^x S_j^x \frac{1}{r_{kj}^{\alpha}} - J_z \sum_{k=1}^{N-1} S_k^z S_{k+1}^z \qquad (1)$$

in order to have an extensive energy ($E \propto N$).
The parameters J, J_z have been chosen in order to have the ground state configuration with all spins along the x direction, with minimal energy

$$E_{min}^x = -\frac{J}{2N^*} \sum_{k=1}^{N} \sum_{j \neq k} \frac{1}{r_{jk}^{\alpha}} \equiv -\frac{JN}{2} \qquad (2)$$

and the latter equivalence defines implicitely N^*, that is

$$N^* = \frac{1}{N}\sum_{k=1}^{N}\sum_{j\neq k}\frac{1}{r_{jk}^{\alpha}}. \tag{3}$$

Choosing now, $J = 2$, and defining the energy per particle $e = E/N$ one has that the ground state energy is $e^x_{min} = -1$ for all α and N.

As a second step we choose a value of J_z such that when all spins are along the z direction, we realize the configuration having the minimal energy with null x-magnetization, i.e.

$$e^z_{min} = \frac{E^z_{min}}{N} = -J_z\frac{N-1}{N} \approx -J_z$$

If now put $J_z = 1/2$ we have two well separated energies : $e^x_{min} = -1$ and $e^z_{min} \simeq -0.5$. For all cases considered here below e^z_{min} is the minimal energy at $M^x = 0$.

Examples of the phase space e, $M^x = (1/N)\sum_k S_k^x$

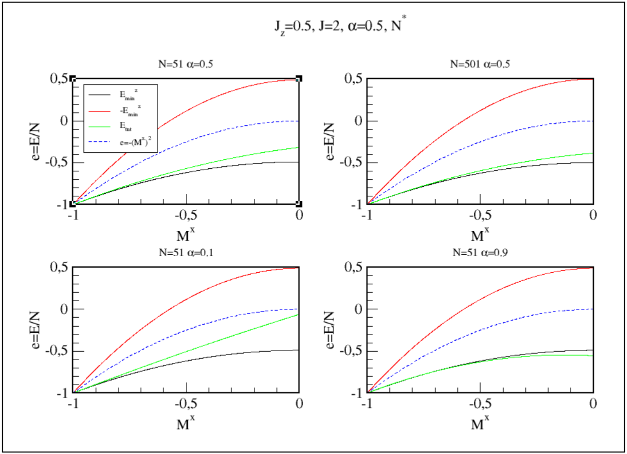


Figure: Examples of the phase space $e = E/N$, M^x for different N and α. Blue curves indicate a particular initial choice for magnetization.

Classical Correlations

In order to study the perturbation spreading we consider the variation in time of the $a = x, y, z$ component of the j-th spin as an effect of varying the $b = x, y, z$ component of the $i \neq j$-th spin as a function of their distance $|j - i|$, see, Metivier, Bachelard, Kastner, PRL 112, 210601 (2014).

$$C_{ji}^{ab}(|i-j|, t) = \left| \frac{\partial S_j^a(t)}{\partial S_i^b(0)} \right| \qquad \text{for} \qquad i \neq j \tag{4}$$

For simplicity we flip the central spin, i.e. we take $i = N/2$ and consider $j \neq i$. Correlations can grow with time, up to the border, namely when $|i - j| = N/2$.

Spreading of Classical Correlations, Long range

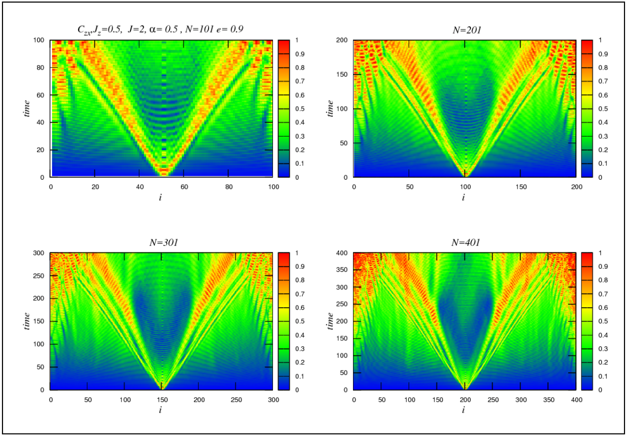


Figure: Classical Correlation C_{zx} as a function of the distance $|i - j|$ and time t. Different panels stand for different N values. Here is $\alpha = 0.5$.

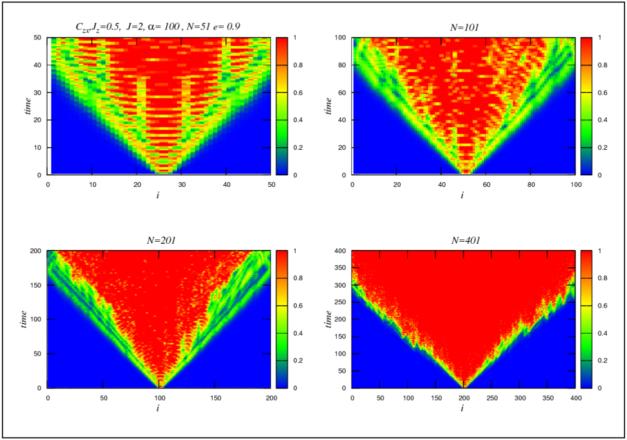


Figure: Classical Correlation C_{zx} as a function of the distance $|i - j|$ and time t. Different panels stand for different N values. Here is $\alpha = 100$.

Measuring correlation out of the linear cone

One can measure the average correlation for a small number of spins (≈ 10) out of the linear cone. One has $\langle C_{zx} \rangle \simeq t^{0.6}/\sqrt{N}$

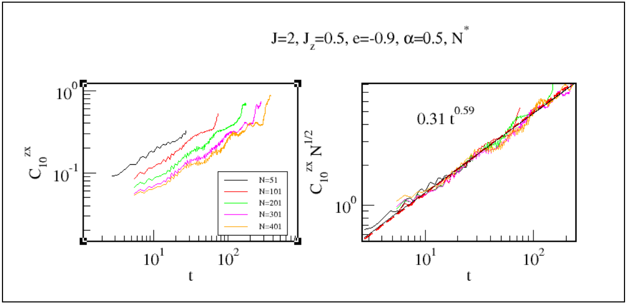


Figure: Average correlation out of the linear cone in a small region (10 spins) for different N values.

Profile of the Sound cone

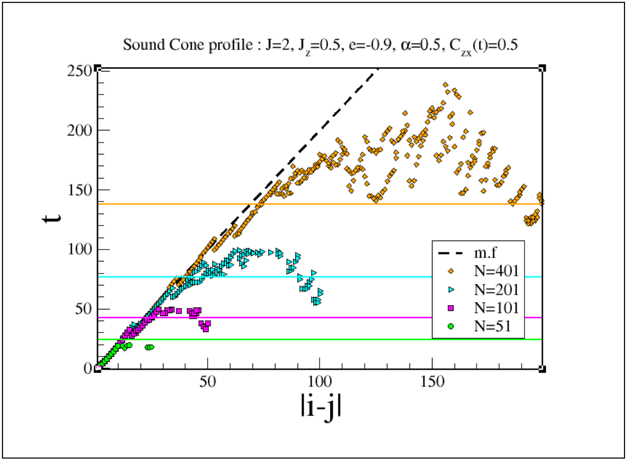


Figure: Profile of the sound cone. It is shown, for any distance $|i - j|$ the first time at which $C_{zx} = 0.5$. Horizonthal lines stand for scaling law.

Thank you for your attention