Neutrino Physics

André de Gouvêa

Northwestern University

Hadron Collider Summer School
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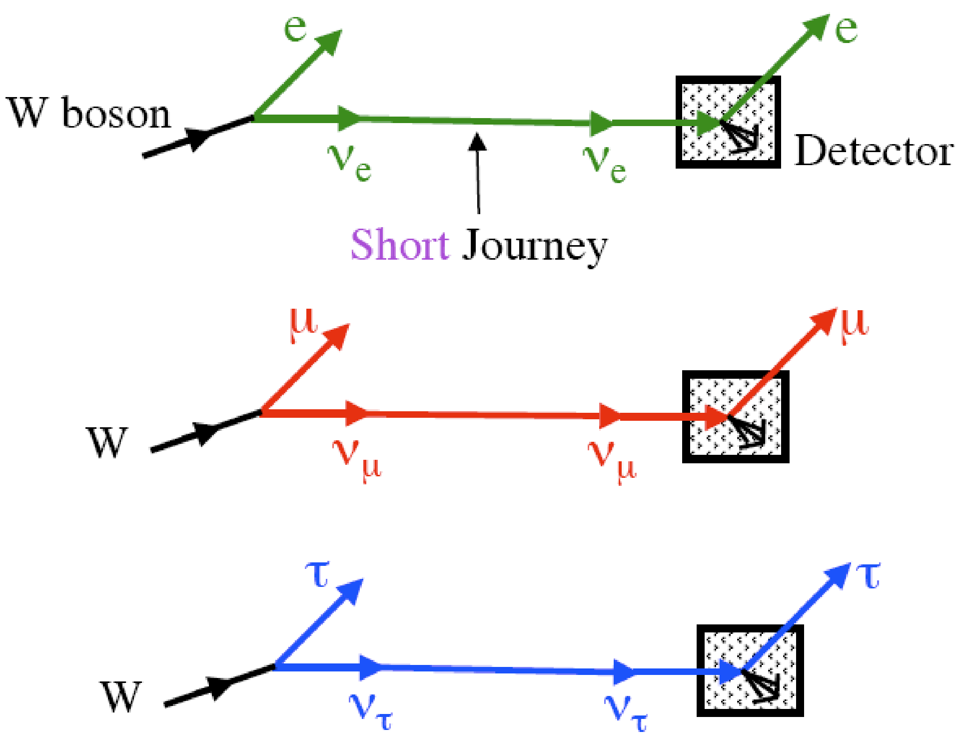
Some Neutrino references (WARNING: Biased Sample)

- "Are There Really Neutrinos? – An Evidential History," Allan Franklin, Perseus Books, 2001. Good discussion of neutrino history.
- A. de Gouvêa, "TASI lectures on neutrino physics," hep-ph/0411274;
- A. de Gouvêa, "Neutrinos have mass: So what?," Mod. Phys. Lett. A **19**, 2799 (2004) [hep-ph/0503086];
- R. N. Mohapatra *et al.*, "Theory of neutrinos: A White paper," Rept. Prog. Phys. **70**, 1757 (2007) [hep-ph/0510213];
- R. N. Mohapatra, A. Yu. Smirnov,"Neutrino Mass and New Physics," Ann. Rev. Nucl. Part. Sci. **56**, 569 (2006) [hep-ph/0603118];
- M. C. Gonzalez-Garcia, M. Maltoni, "Phenomenology with Massive Neutrinos," Phys. Rept. **460**, 1 (2008) [arXiv:0704.1800 [hep-ph]];
- A. Strumia, F. Vissani, "Neutrino masses and mixings," hep-ph/0606054 (2010);
- "The Physics of Neutrinos,"V. Barger, D. Marfatia, K. Whisnant, Princeton University Press (2012);
- "J. Hewett *et al.*, "Fundamental Physics at the Intensity Frontier," arXiv:1205.267;
- A. de Gouvêa *et al.*, "Working Group Report: Neutrinos," arXiv:1310:4340.

What We Knew of Neutrinos: End of the 20th Century



- come in three flavors (see figure);
- interact only via weak interactions ($W^{\pm}, Z^0$);
- have ZERO mass – helicity good quantum number;
- ν_L field describes 2 degrees of freedom:
  - left-handed state ν,
  - right-handed state $\bar{\nu}$ (CPT conjugate);
- neutrinos carry lepton number:
  - $L(\nu) = +1$,
  - $L(\bar{\nu}) = -1$.

2– Neutrino Puzzles – 1960's to 2000's

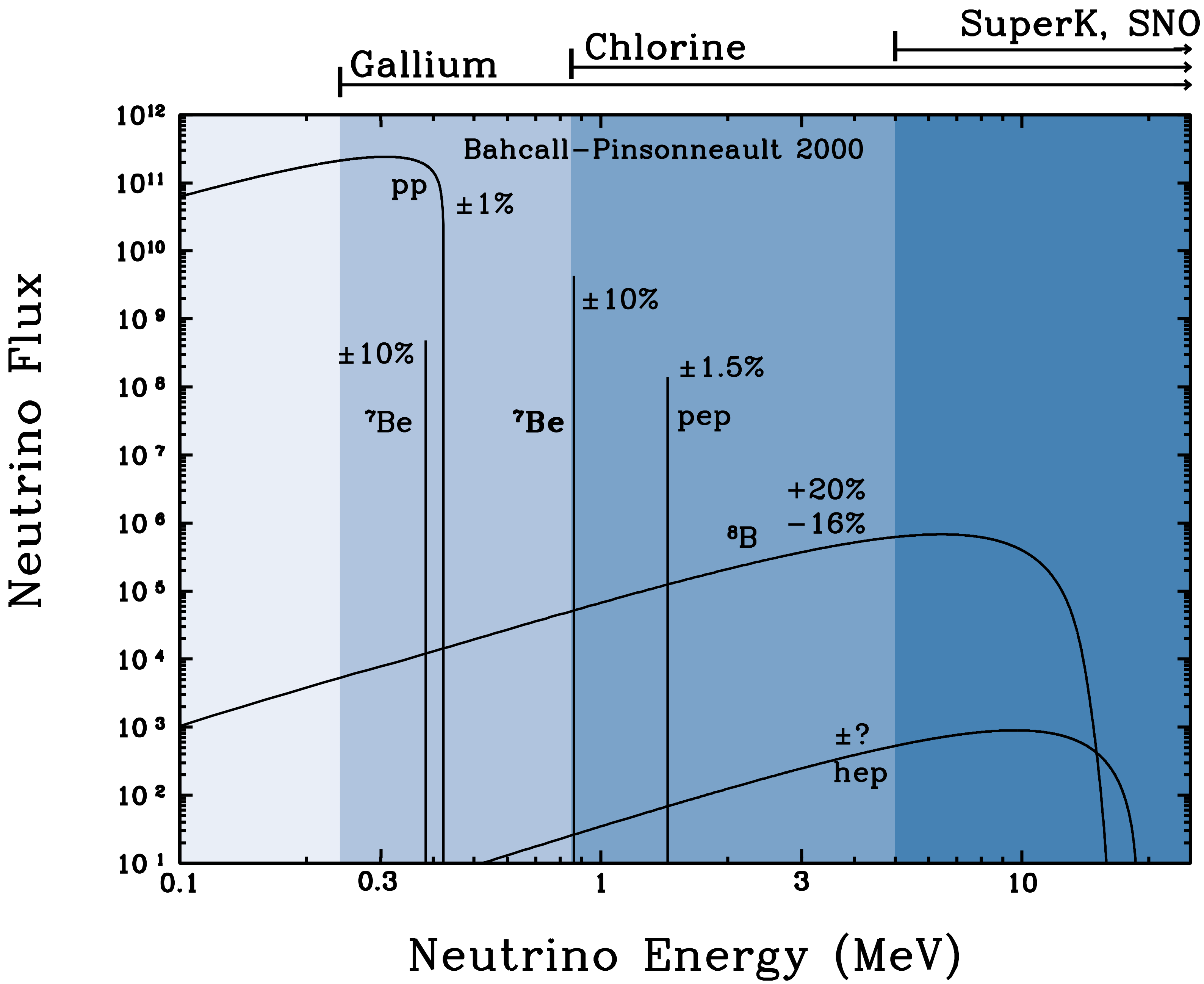
Long baseline neutrino experiments have revealed that neutrinos change flavor after propagating a finite distance, violating the definitions in the previous slide. The rate of change depends on the neutrino energy E_ν and the baseline L.

- $\nu_\mu \to \nu_\tau$ and $\bar{\nu}_\mu \to \bar{\nu}_\tau$ — atmospheric experiments [“indisputable”];
- $\nu_e \to \nu_{\mu,\tau}$ — solar experiments [“indisputable”];
- $\bar{\nu}_e \to \bar{\nu}_{\rm other}$ — reactor neutrinos [“indisputable”];
- $\nu_\mu \to \nu_{\rm other}$ — from accelerator experiments [“indisputable”].

Table 1. Nuclear reactions responsible for producing almost all of the Sun's energy and the different "types" of solar neutrinos (nomenclature): *pp*-neutrinos, *pep*-neutrinos, *hep*-neutrinos, ^{7}Be-neutrinos, and ^{8}B-neutrinos. 'Termination' refers to the fraction of interacting protons that participate in the process.

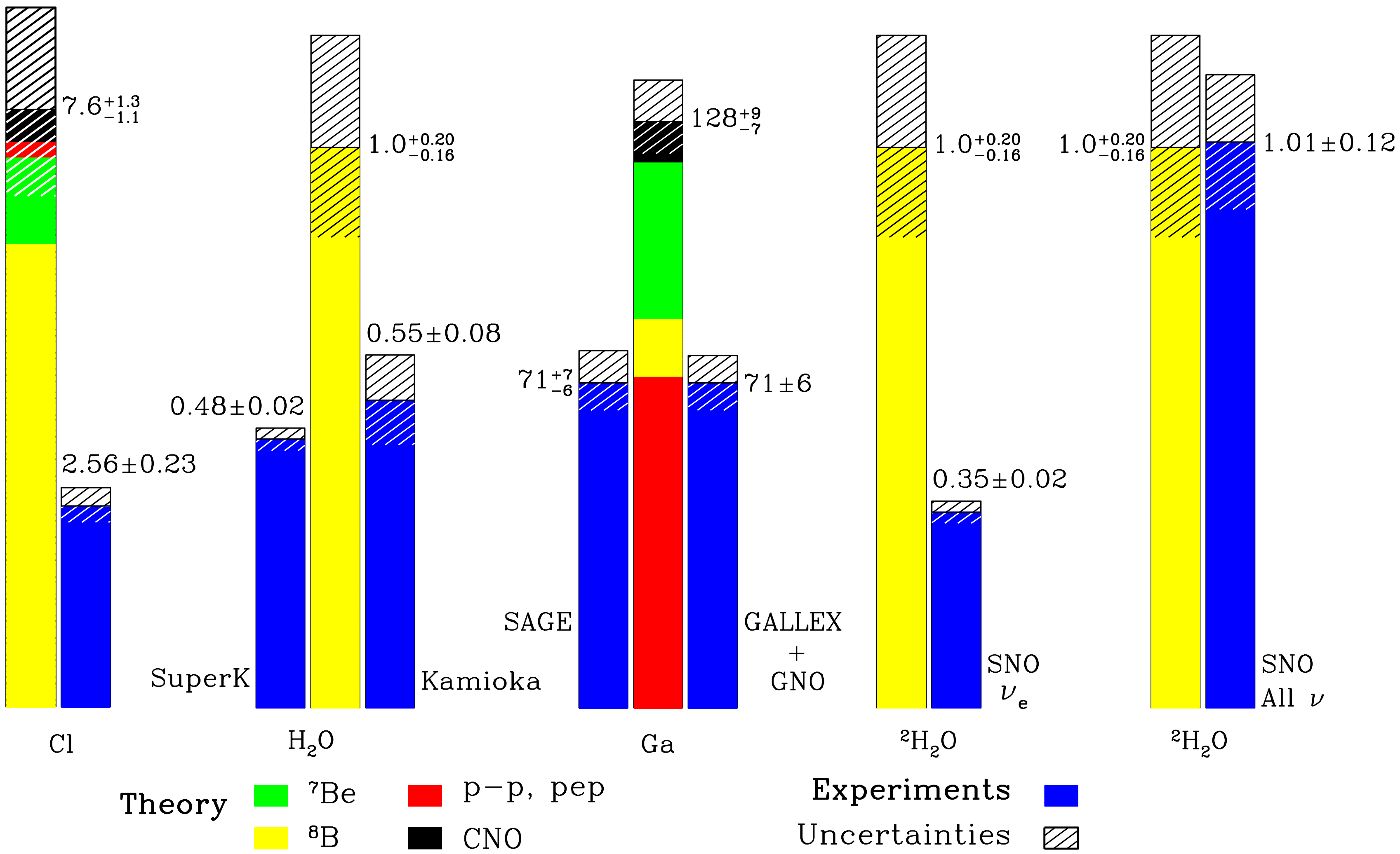
Reaction	Termination (%)	Neutrino Energy (MeV)	Nomenclature
$p+p\rightarrow{}^2\mathrm{H}+e^++\nu_e$	99.96	<0.423	*pp*-neutrinos
$p+e^-+p\rightarrow{}^2\mathrm{H}+\nu_e$	0.044	1.445	*pep*-neutrinos
$^2\mathrm{H}+p\rightarrow{}^3\mathrm{He}+\gamma$	100	–	–
$^3\mathrm{He}+{}^3\mathrm{He}\rightarrow{}^4\mathrm{He}+p+p$	85	–	–
$^3\mathrm{He}+{}^4\mathrm{He}\rightarrow{}^7\mathrm{Be}+\gamma$	15	–	–
$^7\mathrm{Be}+e^-\rightarrow{}^7\mathrm{Li}+\nu_e$	15	0.863(90%) 0.386(10%)	^{7}Be-neutrinos
$^7\mathrm{Li}+p\rightarrow{}^4\mathrm{He}+{}^4\mathrm{He}$		–	–
$^7\mathrm{Be}+p\rightarrow{}^8\mathrm{B}+\gamma$	0.02	–	–
$^8\mathrm{B}\rightarrow{}^8\mathrm{Be}^*+e^++\nu_e$		<15	^{8}B-neutrinos
$^8\mathrm{Be}\rightarrow{}^4\mathrm{He}+{}^4\mathrm{He}$		–	–
$^3\mathrm{He}+p\rightarrow{}^4\mathrm{He}+e^++\nu_e$	0.00003	<18.8	*hep*-neutrinos

Note: Adapted from Ref. 12. Please refer to Ref. 12 for a more detailed explanation.

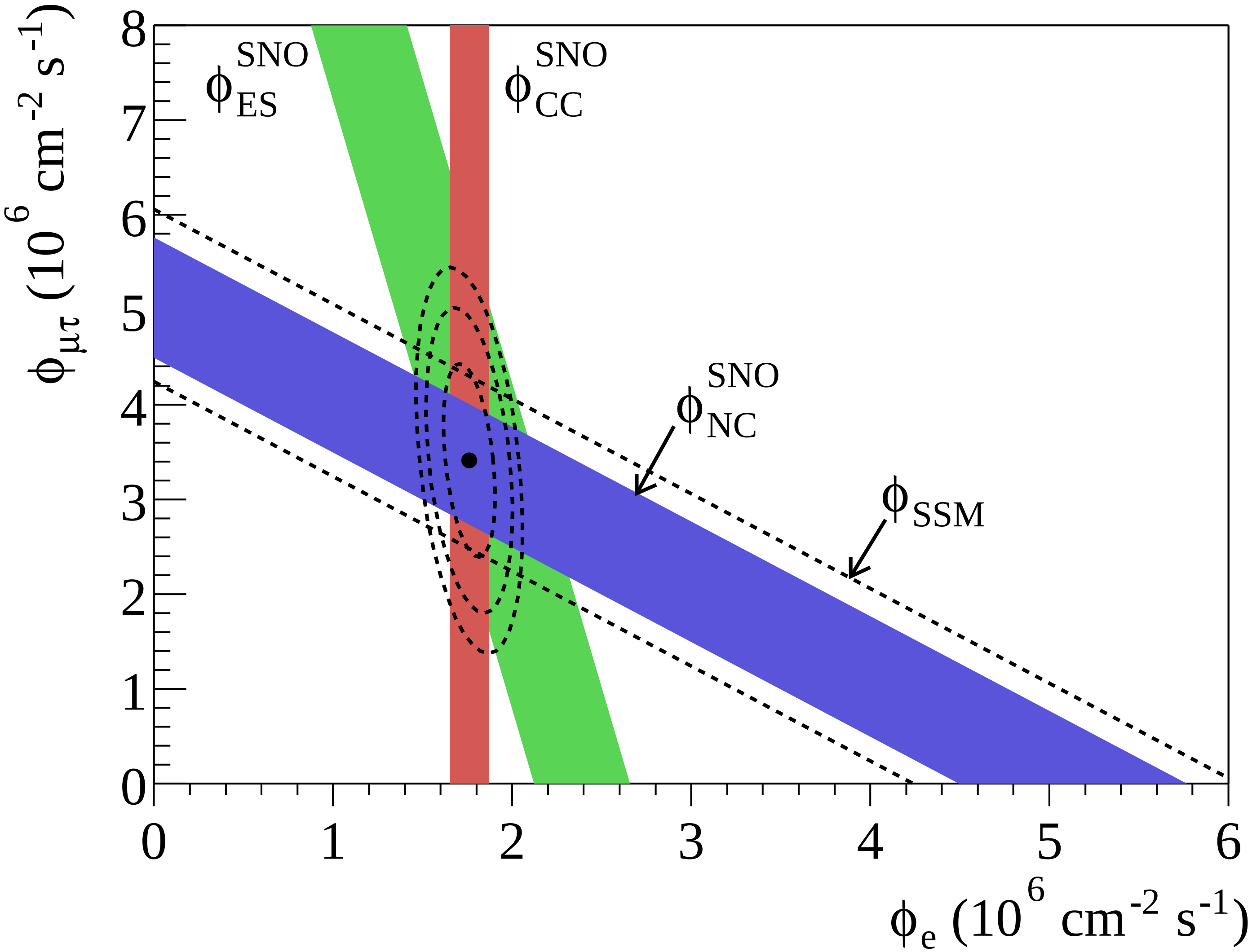

SuperK, SNO
Chlorine
Gallium
Bahcall−Pinsonneault 2000
pp
±1%
±10%
^{7}Be
±10%
^{7}Be
±1.5%
pep
+20%
−16%
^{8}B
±?
hep
10^{12}
10^{11}
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
10^{5}
10^{4}
10^{3}
10^{2}
10^{1}
0.1
0.3
1
3
10
Neutrino Flux
Neutrino Energy (MeV)

Total Rates: Standard Model vs. Experiment

Bahcall–Pinsonneault 2000



The SNO Experiment: conclusive evidence for flavor change



SNO Measures:

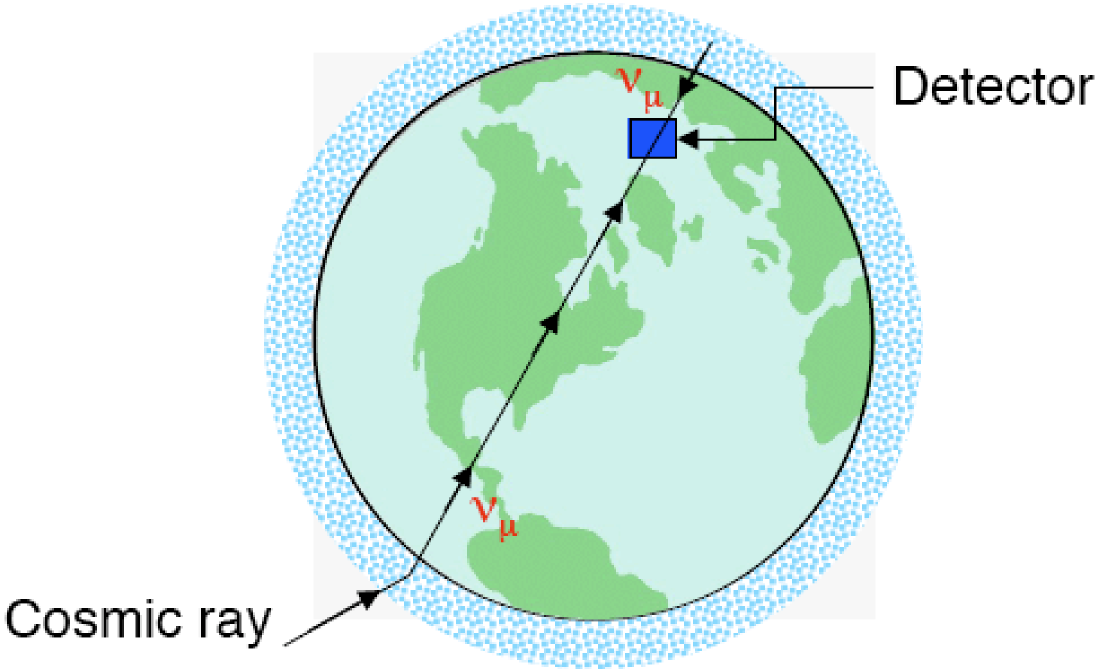
$[CC]$ $\nu_e + {}^2H \to p + p + e^-$

$[ES]$ $\nu + e^- \to \nu + e^-$

$[NC]$ $\nu + {}^2H \to p + n + \nu$

different reactions sensitive to different neutrino flavors.

Atmospheric Neutrinos



Isotropy of the $\gtrsim 2$ GeV cosmic rays + Gauss' Law + No ν_μ disappearance

$$\Rightarrow \quad \frac{\phi_{\nu_\mu}(\mathrm{Up})}{\phi_{\nu_\mu}(\mathrm{Down})} = 1 \ .$$

But Super-Kamiokande finds for $E_\nu > 1.3$ GeV

$$\frac{\phi_{\nu_\mu}(\mathrm{Up})}{\phi_{\nu_\mu}(\mathrm{Down})} = 0.54 \pm 0.04 \ .$$

UP $\neq$ DOWN – neutrinos can tell time! $\rightarrow$ neutrinos have mass.

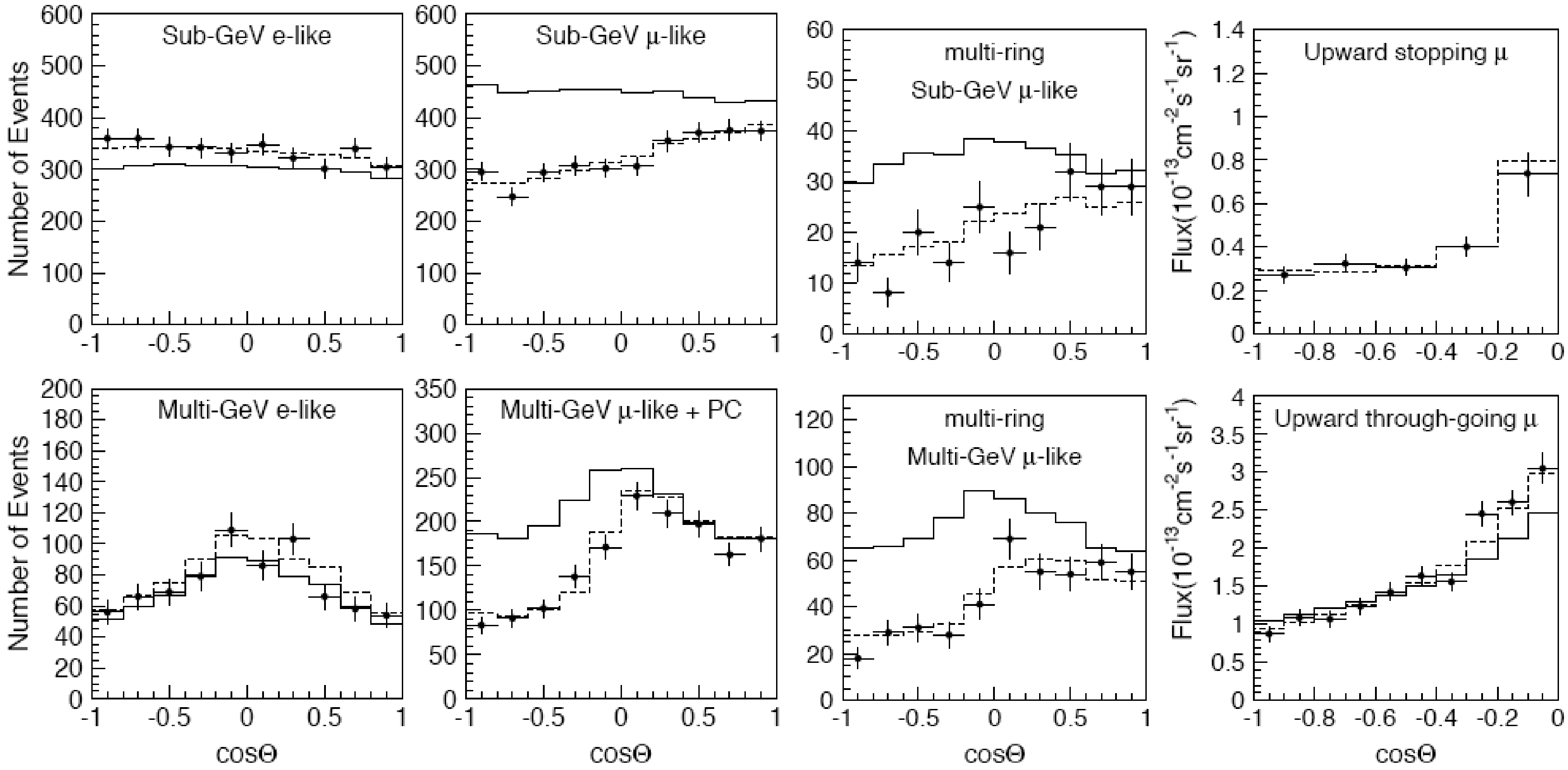


Figure 4. Zenith angle distribution for fully-contained single-ring e-like and μ-like events, multi-ring μ-like events, partially contained events and upward-going muons. The points show the data and the solid lines show the Monte Carlo events without neutrino oscillation. The dashed lines show the best-fit expectations for $\nu_\mu \leftrightarrow \nu_\tau$ oscillations. From M. Ishitsuka [Super-Kamiokande Collaboration], hep-ex/0406076.

3 - Mass-Induced Neutrino Flavor Oscillations

Neutrino Flavor change can arise out of several different mechanisms. The simplest one is to appreciate that, once **neutrinos have mass**, **leptons can mix**. This turns out to be the correct mechanism (certainly the dominant one), and **only** explanation that successfully explains **all** long-baseline data consistently.

Neutrinos with a well defined mass:

$$\nu_1, \nu_2, \nu_3, \ldots \qquad \text{with masses} \;\; m_1, m_2, m_3, \ldots$$

How do these states (neutrino mass eigenstates) relate to the neutrino flavor eigenstates $(\nu_e, \nu_\mu, \nu_\tau)$?

$$\nu_\alpha = U_{\alpha i}\nu_i \qquad \alpha = e, \mu, \tau, \quad i = 1, 2, 3$$

U is a unitary mixing matrix. I'll talk more about it later.

The Propagation of Massive Neutrinos

Neutrino mass eigenstates are eigenstates of the free-particle Hamiltonian:

$$|\nu_i\rangle = e^{-iE_it}|\nu_i\rangle, \qquad E_i^2 - |\vec{p}_i|^2 = m_i^2$$

The neutrino flavor eigenstates are linear combinations of ν_i's, say:

$$\begin{aligned} |\nu_e\rangle &= \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle. \\ |\nu_\mu\rangle &= -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle. \end{aligned}$$

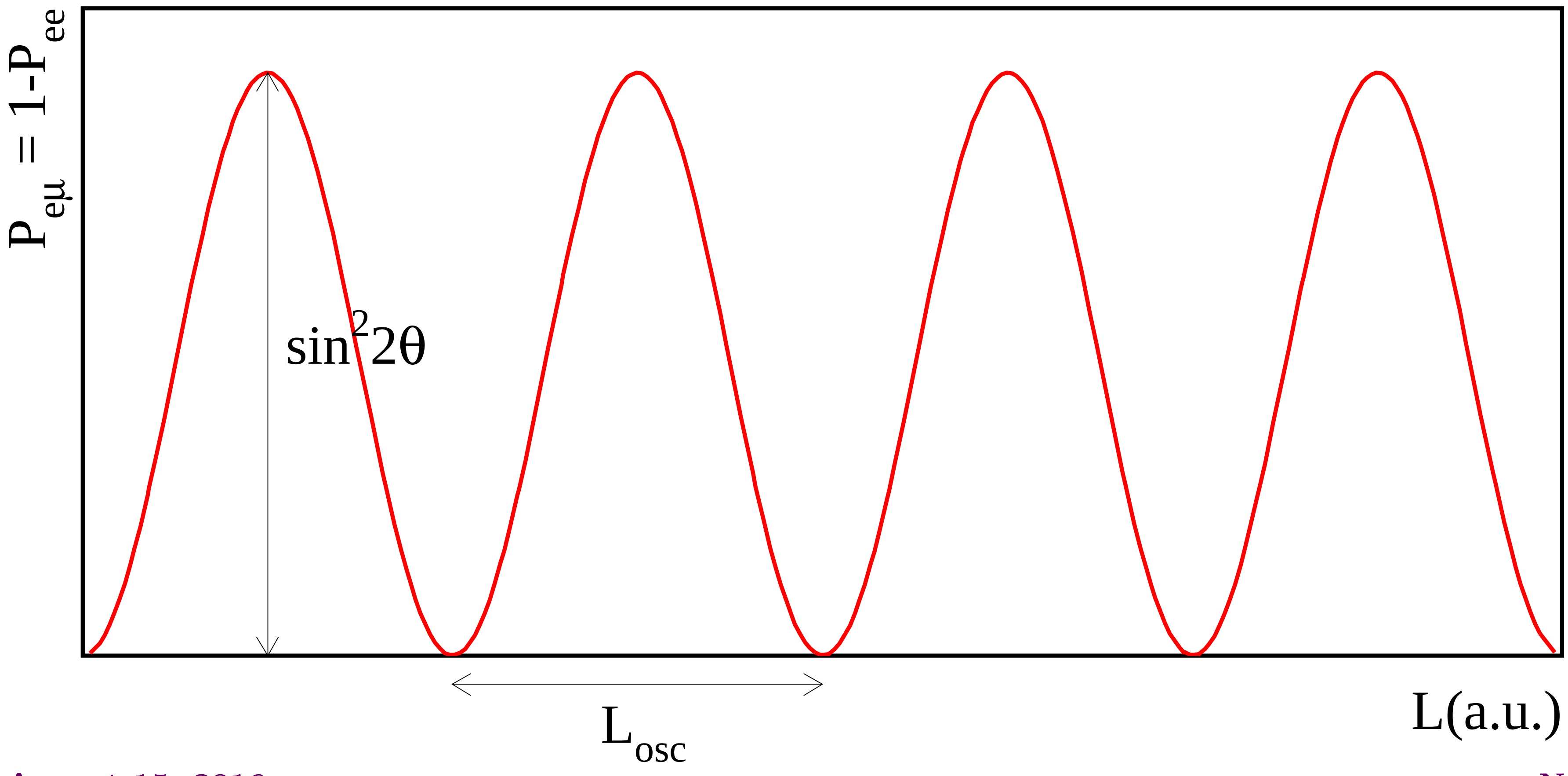
If this is the case, a state produced as a ν_e evolves in vacuum into

$$|\nu(t,\vec{x})\rangle = \cos\theta e^{-ip_1x}|\nu_1\rangle + \sin\theta e^{-ip_2x}|\nu_2\rangle.$$

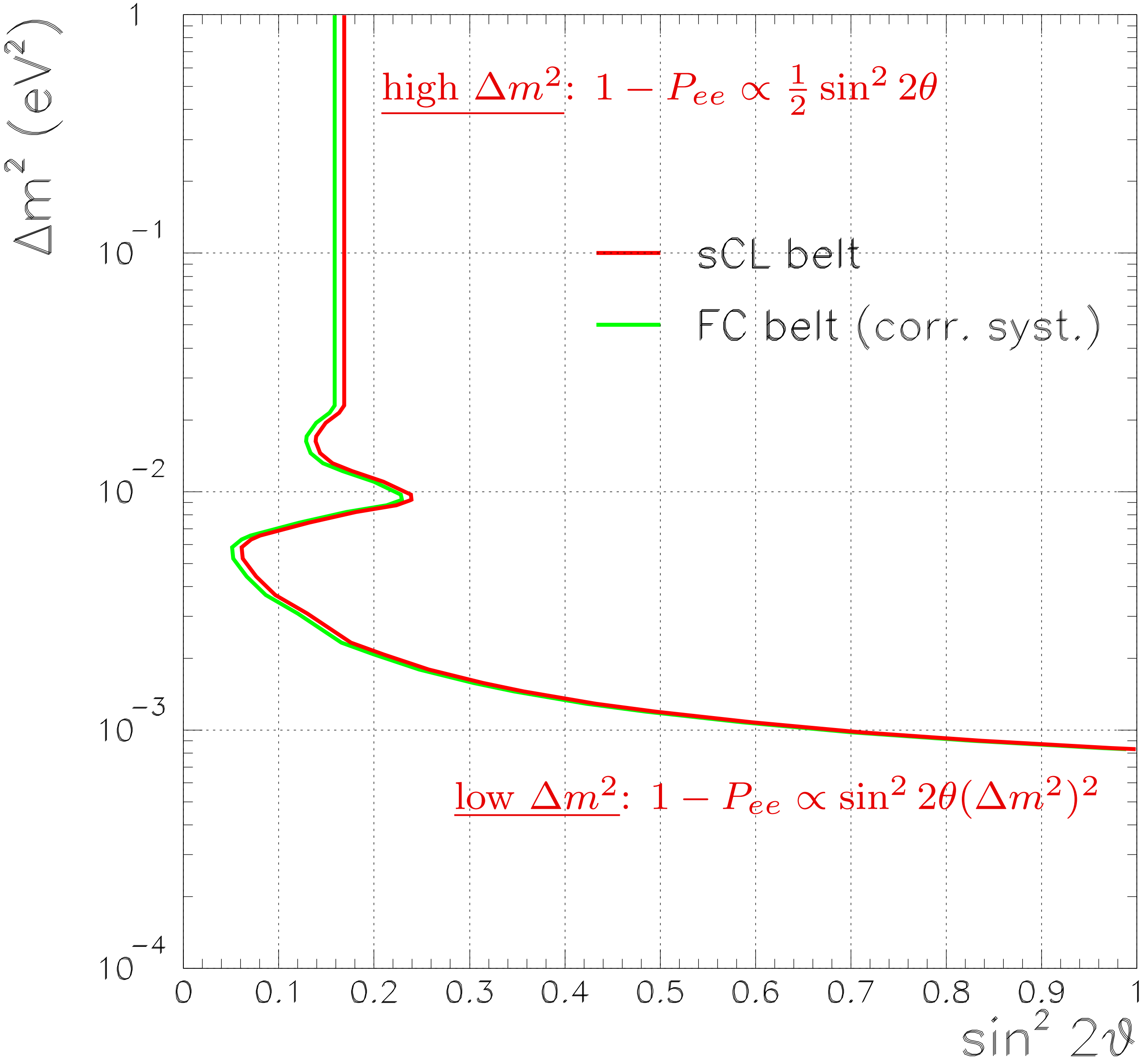
It is trivial to compute $P_{e\mu}(L) \equiv |\langle\nu_\mu|\nu(t,z=L)\rangle|^2$. It is just like a two-level system from basic undergraduate quantum mechanics! In the ultrarelativistic limit (always a good bet), $t \simeq L$, $E_i - p_{z,i} \simeq (m_i^2)/2E_i$, and

$$\boxed{P_{e\mu}(L) = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right)}$$

oscillation parameters: $\begin{cases} \pi \frac{L}{L_{\rm osc}} \equiv \frac{\Delta m^2 L}{4E} = 1.267 \left(\frac{L}{\rm km}\right) \left(\frac{\Delta m^2}{\rm eV^2}\right) \left(\frac{\rm GeV}{E}\right) \\ \text{amplitude } \sin^2 2\theta \end{cases}$



CHOOZ experiment



$$P_{ee} = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

result: $1 - P_{ee} < 0.05$

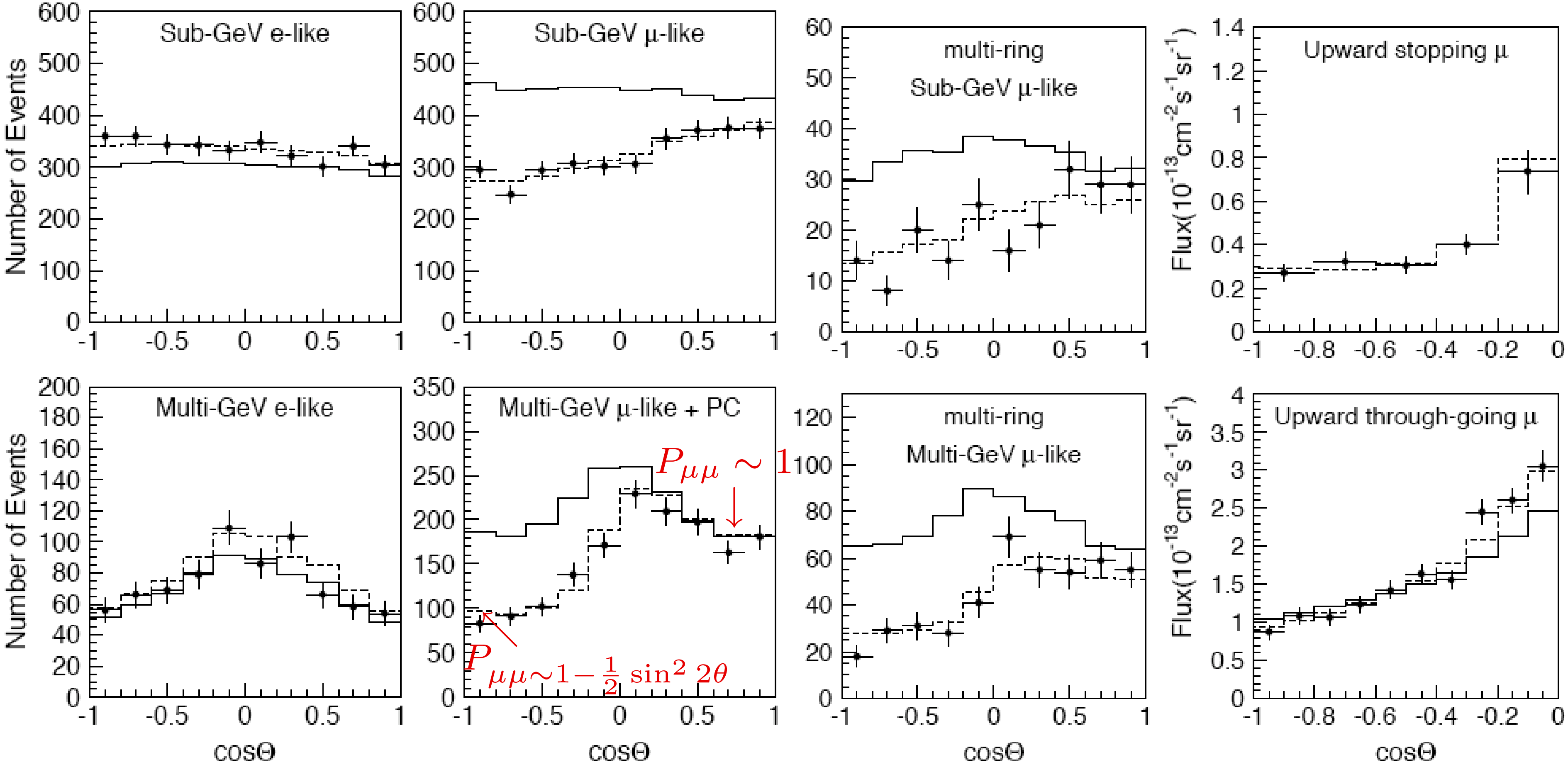
There is a long (and oftentimes confused and confusing) history behind this derivation and several others. A comprehensive discussion can be found, for example, in

E.K. Akhmedov, A. Yu. Smirnov, 0905.1903 [hep-ph]

In a nutshell, neutrino oscillations as described above occur whenever

- Neutrino Production and Detection are Coherent → cannot "tell" ν_1 from ν_2 from ν_3 but "see" ν_e or ν_μ or ν_τ.
- Decoherence effects due to wave-packet separation are negligible → baseline not too long that different "velocity" components of the neutrino wave-packet have time to physically separate.
- The energy released in production and detection is large compared to the neutrino mass → so we can assign all of the effect to the neutrino propagation, independent from the production process. Also assures ultra-relativistic approximation good.

$P_{\mu\mu} = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$ Works great for $\sin^2 2\theta \sim 1$ and $\Delta m^2 \sim 10^{-3}$ eV^2



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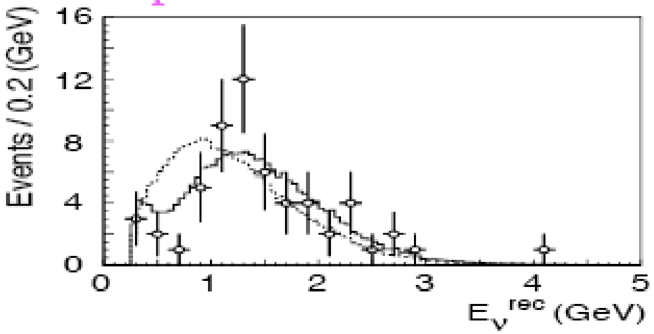
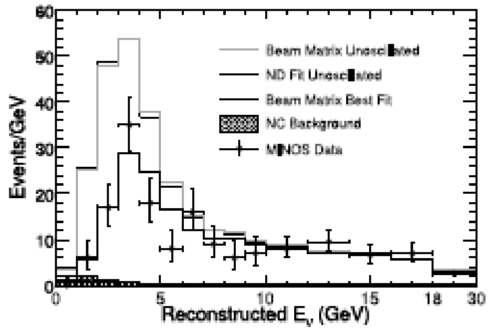
K2K	ν_μ at KEK	SK	L=250 km
MINOS	ν_μ at Fermilab	Soundan	L=735 km
Opera/Icarus	ν_μ at CERN	Gran Sasso	L=740 km

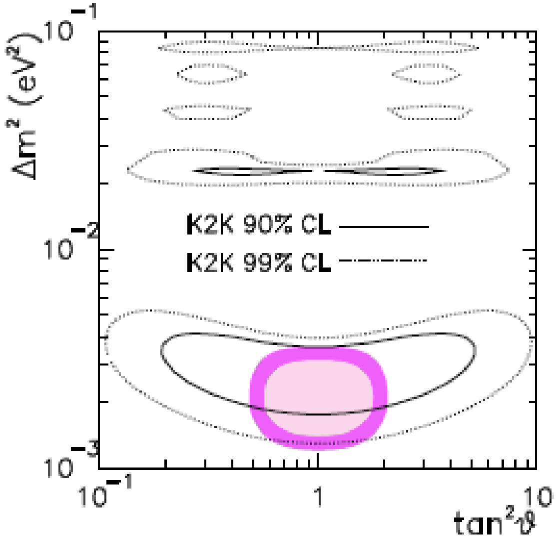
K2K 2004: spectral distortion



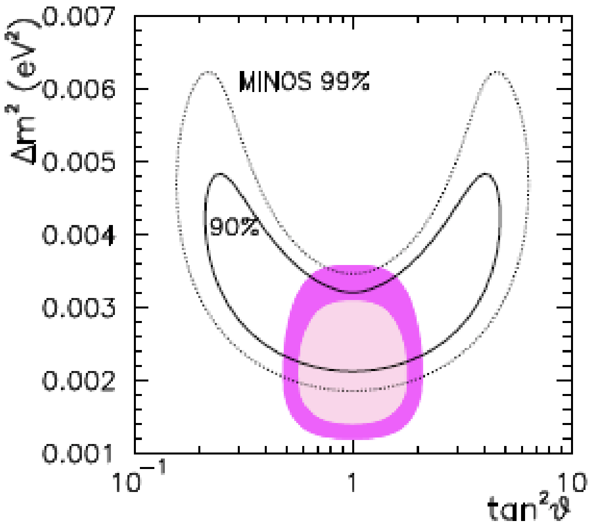
MINOS 2006: spectral distortion



Confirmation of ATM oscillations



Confirmation of ATM oscillations



[Gonzalez-Garcia, PASI 2006]

Matter Effects

The neutrino propagation equation, in the ultra-relativistic approximation, can be re-expressed in the form of a Shrödinger-like equation. In the mass basis:

$$i\frac{\mathrm{d}}{\mathrm{d}L}|\nu_i\rangle = \frac{m_i^2}{2E}|\nu_i\rangle,$$

up to a term proportional to the identity. In the weak/flavor basis

$$i\frac{\mathrm{d}}{\mathrm{d}L}|\nu_\beta\rangle = U_{\beta i}\frac{m_i^2}{2E}U^\dagger_{i\alpha}|\nu_\alpha\rangle.$$

In the 2×2 case,

$$i\frac{\mathrm{d}}{\mathrm{d}L}\left(\begin{array}{c}|\nu_e\rangle\\ |\nu_\mu\rangle\end{array}\right) = \frac{\Delta m^2}{2E}\left(\begin{array}{cc}\sin^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \cos^2\theta\end{array}\right)\left(\begin{array}{c}|\nu_e\rangle\\ |\nu_\mu\rangle\end{array}\right),$$

(again, up to additional terms proportional to the 2×2 identity matrix).

Fermi Lagrangian, after a Fiertz rearrangement of the charged-current terms:

$$\mathrm{L} \supset \bar{\nu}_{eL} i\partial_\mu \gamma^\mu \nu_{eL} - 2\sqrt{2} G_F \left(\bar{\nu}_{eL}\gamma^\mu \nu_{eL}\right)\left(\bar{e}_L \gamma_\mu e_L\right) + \ldots$$

Equation of motion for one electron neutrino state in the presence of a non-relativistic electron background, in the rest frame of the electrons:

$$\langle \bar{e}_L \gamma_\mu e_L \rangle = \delta_{\mu 0} \frac{N_e}{2}$$

where $N_e \equiv e^\dagger e$ is the average electron number density (at rest, hence $\delta_{\mu 0}$ term). Factor of $1/2$ from the "left-handed" half.

Dirac equation for a one neutrino state inside a cold electron "gas" is (ignore neutrino mass)

$$(i\partial^\mu \gamma_\mu - \sqrt{2} G_F N_e \gamma_0)|\nu_e\rangle = 0.$$

In the ultrarelativistic limit, (plus $\sqrt{2}G_F N_e \ll E$), dispersion relation is

$$E \simeq |\vec{p}| \pm \sqrt{2} G_F N_e, \qquad + \text{ for } \nu, \quad - \text{ for } \bar{\nu}$$

$$i\frac{\mathrm{d}}{\mathrm{d}L}\left(\begin{array}{c}|\nu_e\rangle\\|\nu_\mu\rangle\end{array}\right)=\left[\frac{\Delta m^2}{2E}\left(\begin{array}{cc}\sin^2\theta & \cos\theta\sin\theta\\ \cos\theta\sin\theta & \cos^2\theta\end{array}\right)+\left(\begin{array}{cc}A & 0\\ 0 & 0\end{array}\right)\right]\left(\begin{array}{c}|\nu_e\rangle\\|\nu_\mu\rangle\end{array}\right),$$

$A=\pm\sqrt{2}G_FN_e$ (+ for neutrinos, − for antineutrinos).

Note: Similar effect from neutral current interactions common to all (active) neutrino species → proportional to the identity.

In general, this is hard to solve, as A is a function of L: two-level non-relativistc quantum mechanical system in the presence of time dependent potential.

In some cases, however, the solution is rather simple.

Constant A: good approximation for neutrinos propagating through matter inside the Earth [exception: neutrinos that see Earth's internal structure (the crust, the mantle, the outer core, the inner core)]

$$i\frac{\mathrm{d}}{\mathrm{d}L}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} A & \Delta/2\sin 2\theta \\ \Delta/2\sin 2\theta & \Delta\cos 2\theta \end{pmatrix}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}, \quad \Delta \equiv \Delta m^2/2E.$$

$$P_{e\mu} = \sin^2 2\theta_M \sin^2\left(\frac{\Delta_M L}{2}\right),$$

where

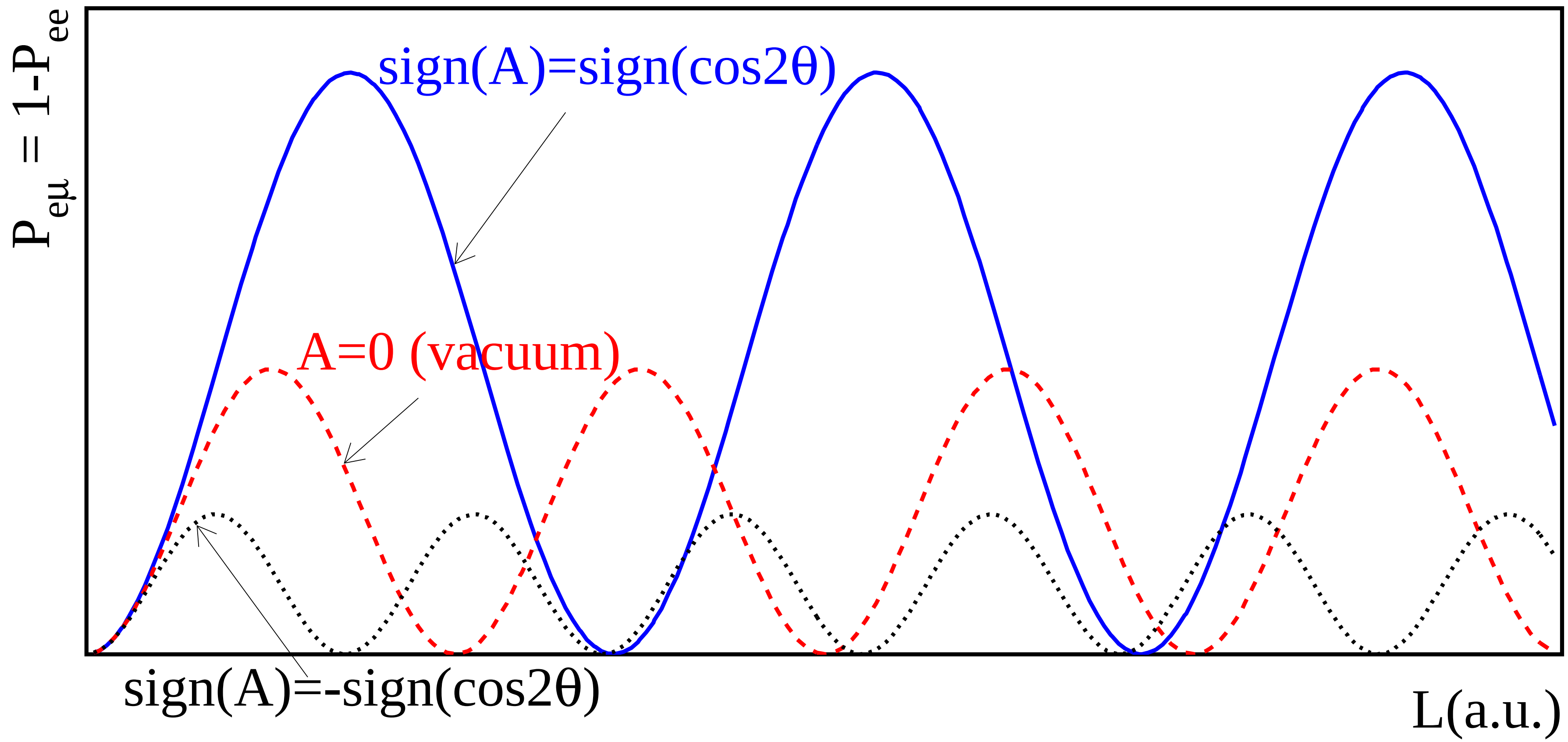
$$\begin{aligned} \Delta_M &= \sqrt{(A - \Delta\cos 2\theta)^2 + \Delta^2\sin^2 2\theta}, \\ \Delta_M \sin 2\theta_M &= \Delta\sin 2\theta, \\ \Delta_M \cos 2\theta_M &= A - \Delta\cos 2\theta. \end{aligned}$$

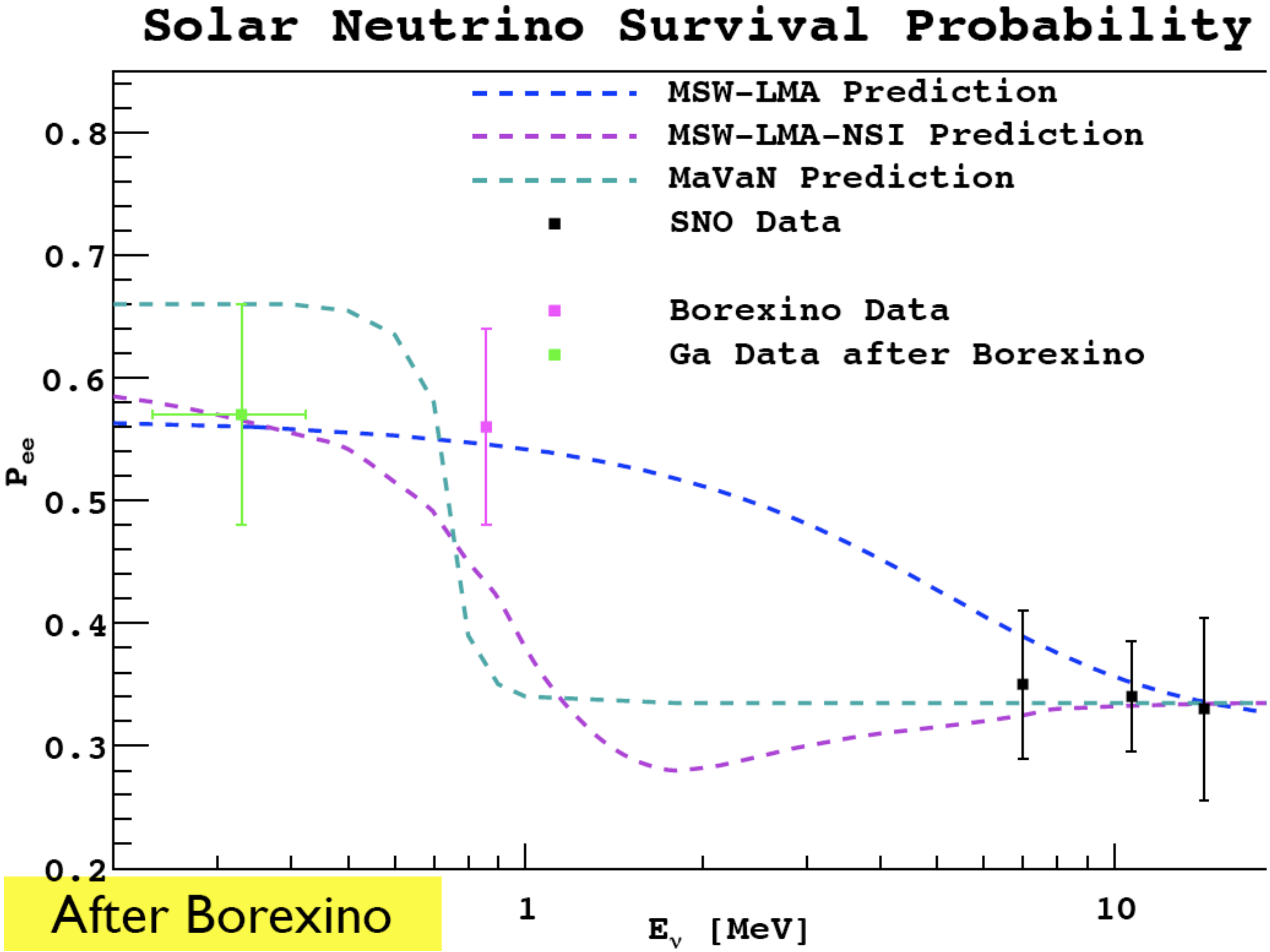
The presence of matter affects neutrino and antineutrino oscillation differently.

Nothing wrong with this: CPT-theorem relates the propagation of neutrinos in an electron background to the propagation of antineutrinos in a positron background.

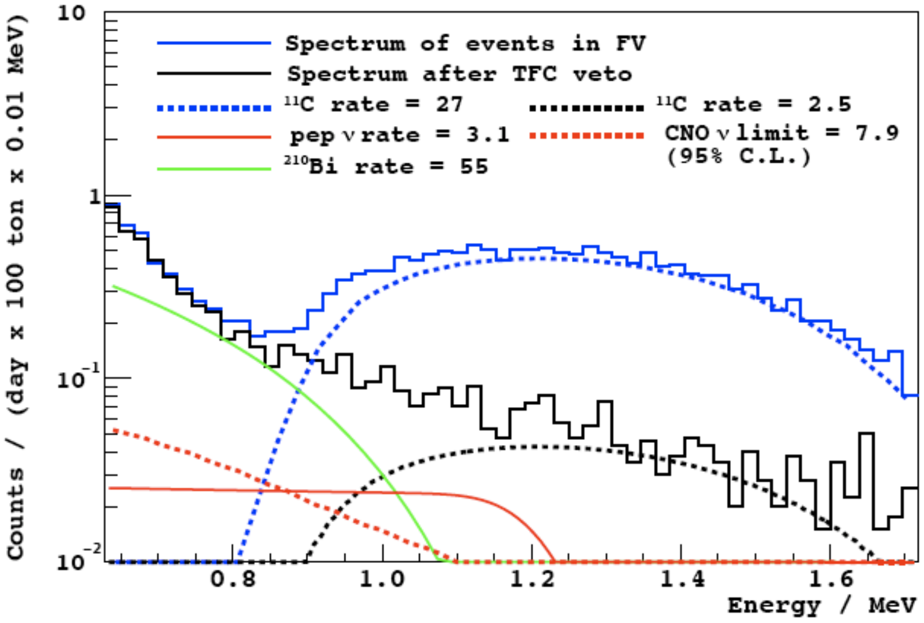
Enlarged parameter space in the presence of matter effects.

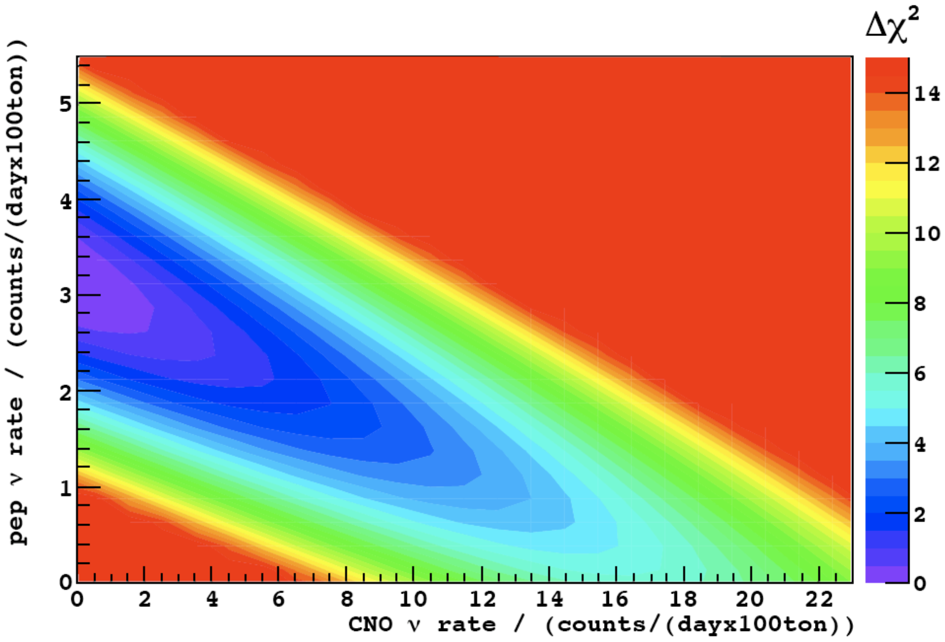
For example, can tell whether $\cos 2\theta$ is positive or negative.

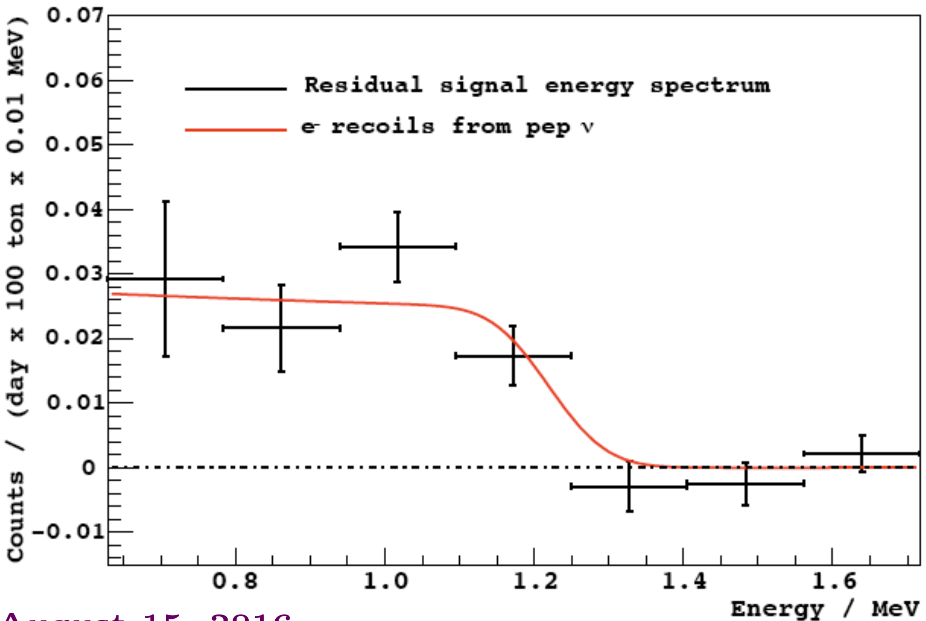


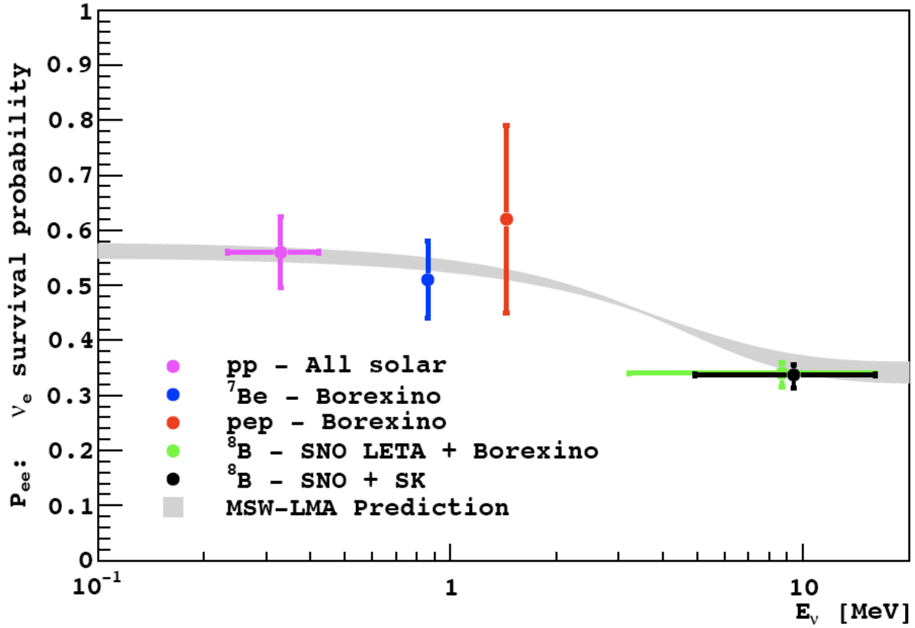

Solar Neutrino Survival Probability
MSW-LMA Prediction
MSW-LMA-NSI Prediction
MaVaN Prediction
SNO Data
Borexino Data
Ga Data after Borexino
P_{ee}
0.8
0.7
0.6
0.5
0.4
0.3
0.2
1
10
E_ν [MeV]
After Borexino

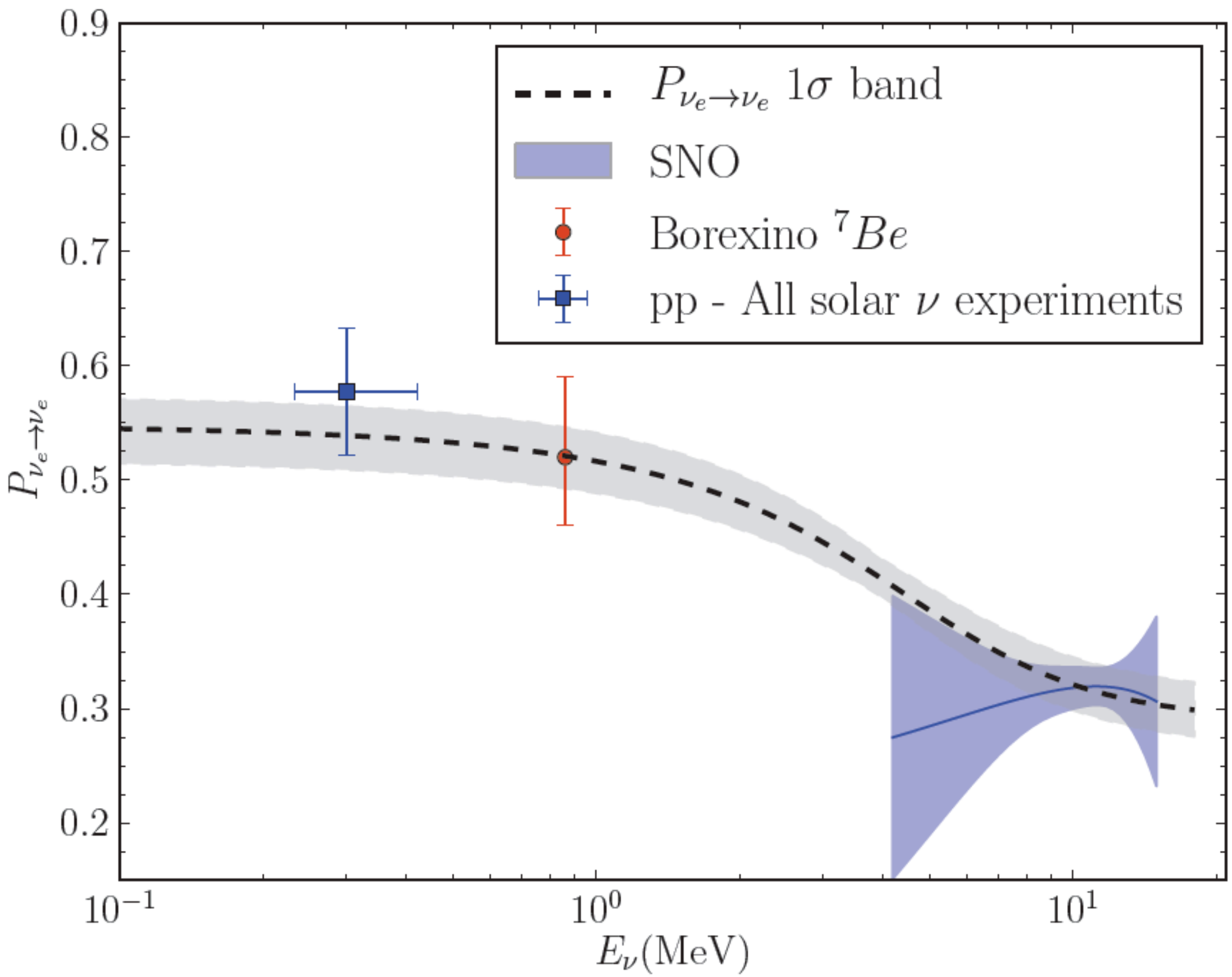
Borexino, 1110.3230

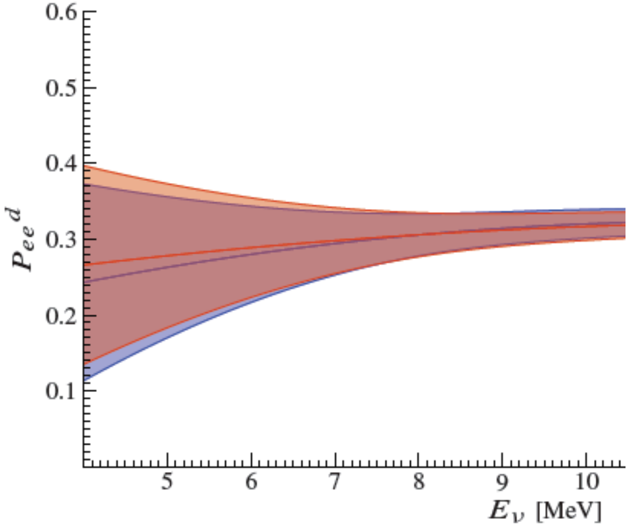


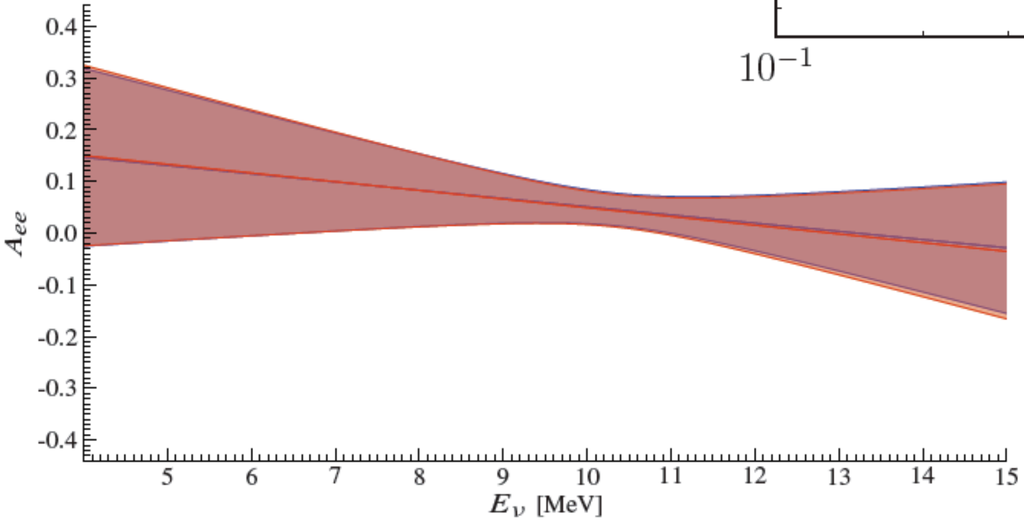










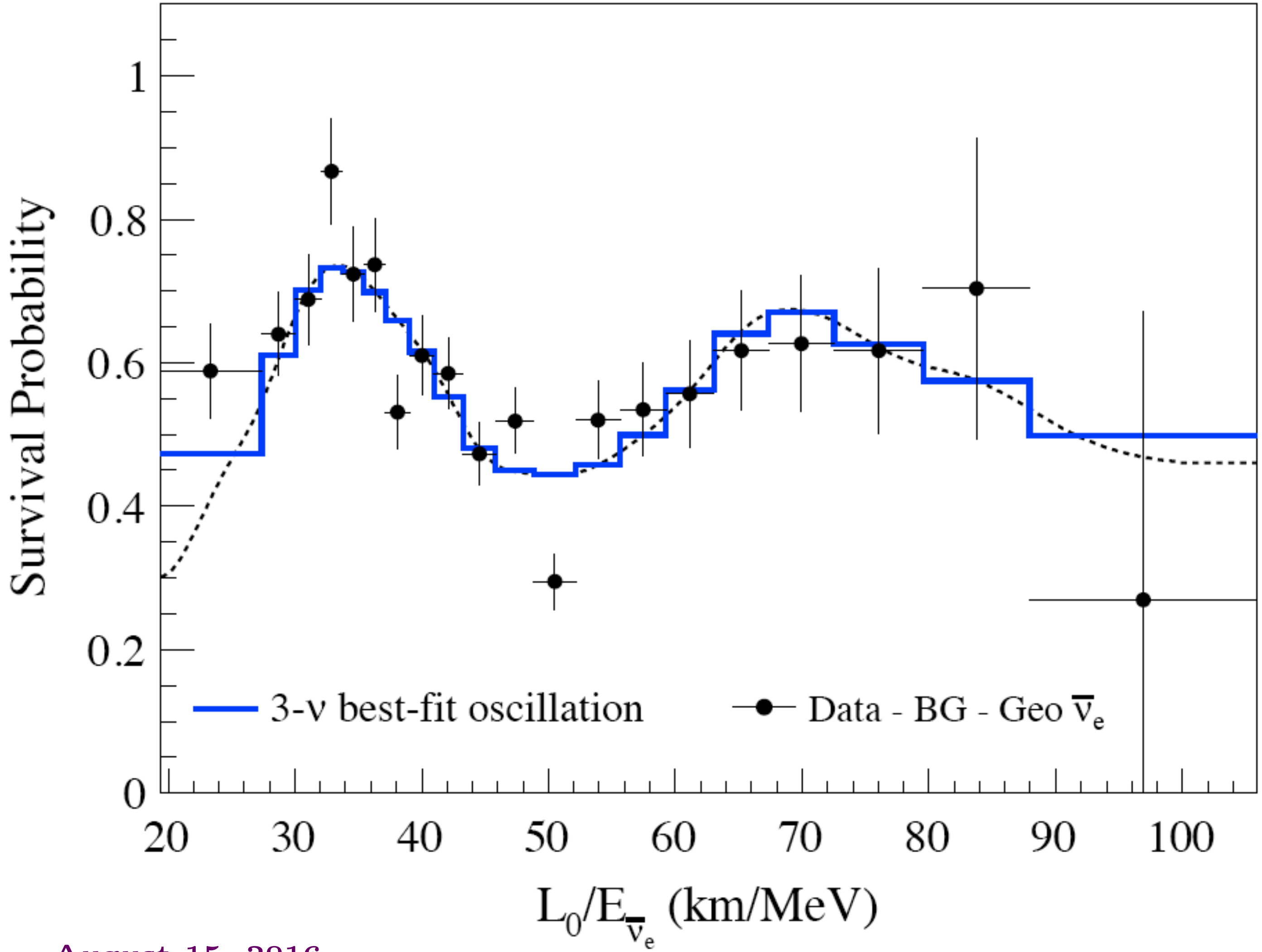


“Final” SNO results, 1109.0763

Solar oscillations confirmed by Reactor experiment: KamLAND

[arXiv:1303.4667]

$$\text{phase} = 1.27\left(\frac{\Delta m^2}{5\times10^{-5}\ \text{eV}^2}\right)\left(\frac{5\ \text{MeV}}{E}\right)\left(\frac{L}{100\ \text{km}}\right)$$

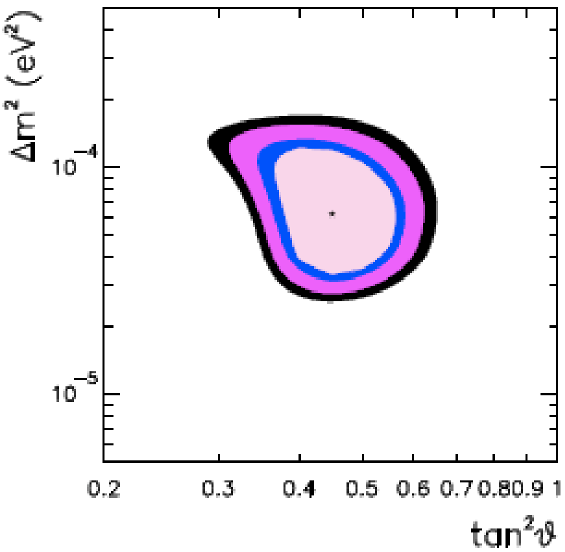


$$P_{ee} = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

oscillatory behavior!

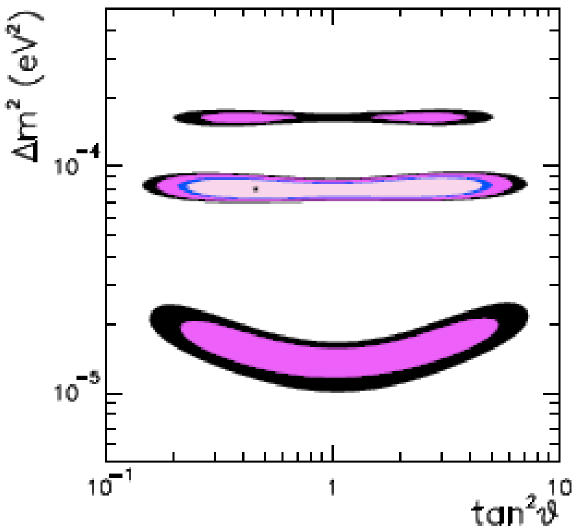
Solar

$\nu_e \to \nu_{\rm active}$

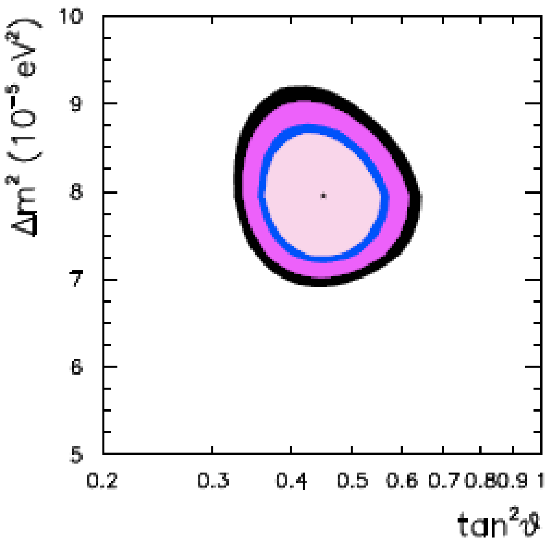


+ KamLAND

$\bar{\nu}_e \not\to \bar{\nu}_e$



ν_e oscillation parameters compatible with $\overline{\nu}_e$: *Sensible* to assume CPT: $P_{ee} = P_{\bar{e}\bar{e}}$

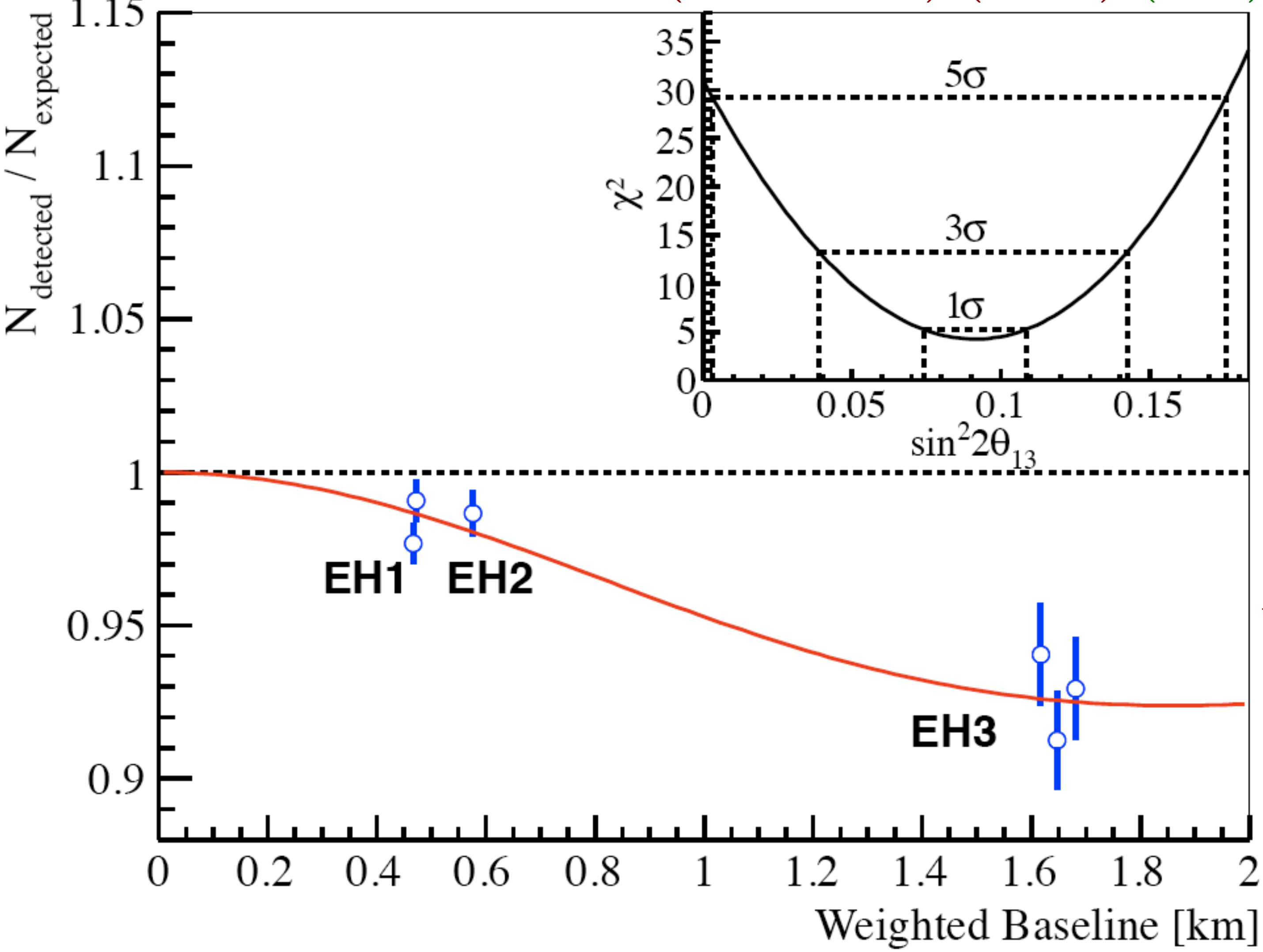


$$\Delta m^2_\odot = (8^{+0.4}_{-0.5}) \times 10^{-5}\ \mathrm{eV}^2 \quad (1\sigma)$$
$$\tan^2\theta_\odot = 0.45^{+0.05}_{-0.05}$$

[Gonzalez-Garcia, PASI 2006]

Atmospheric Oscillations in the Electron Sector: Daya Bay, RENO, Double Chooz

$$\text{phase} = 0.64\left(\frac{\Delta m^2}{2.5\times10^{-3}~\text{eV}^2}\right)\left(\frac{5~\text{MeV}}{E}\right)\left(\frac{L}{1~\text{km}}\right)$$



$$P_{ee} = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

Summarizing:

Both the solar and atmospheric puzzles can be properly explained in terms of two-flavor neutrino oscillations:

- **solar**: $\nu_e \leftrightarrow \nu_a$ (linear combination of ν_μ and ν_τ): $\Delta m^2 \sim 10^{-4}$ eV^2, $\sin^2\theta \sim 0.3$.
- **atmospheric**: $\nu_\mu \leftrightarrow \nu_\tau$: $\Delta m^2 \sim 10^{-3}$ eV^2, $\sin^2\theta \sim 0.5$ ("maximal mixing").
- **short-baseline reactors:** $\nu_e \leftrightarrow \nu_a$ (linear combination of ν_μ and ν_τ): $\Delta m^2 \sim 10^{-3}$ eV^2, $\sin^2\theta \sim 0.02$.

Putting it all together – 3 flavor mixing:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{e\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Definition of neutrino mass eigenstates (who are ν_1, ν_2, ν_3?):

- $m_1^2 < m_2^2$ — $\Delta m^2_{13} < 0$ – Inverted Mass Hierarchy
- $m_2^2 - m_1^2 \ll |m_3^2 - m_{1,2}^2|$ — $\Delta m^2_{13} > 0$ – Normal Mass Hierarchy

$$\tan^2\theta_{12} \equiv \frac{|U_{e2}|^2}{|U_{e1}|^2}; \quad \tan^2\theta_{23} \equiv \frac{|U_{\mu 3}|^2}{|U_{\tau 3}|^2}; \quad U_{e3} \equiv \sin\theta_{13} e^{-i\delta}$$

[For a detailed discussion see AdG, Jenkins, PRD78, 053003 (2008)]

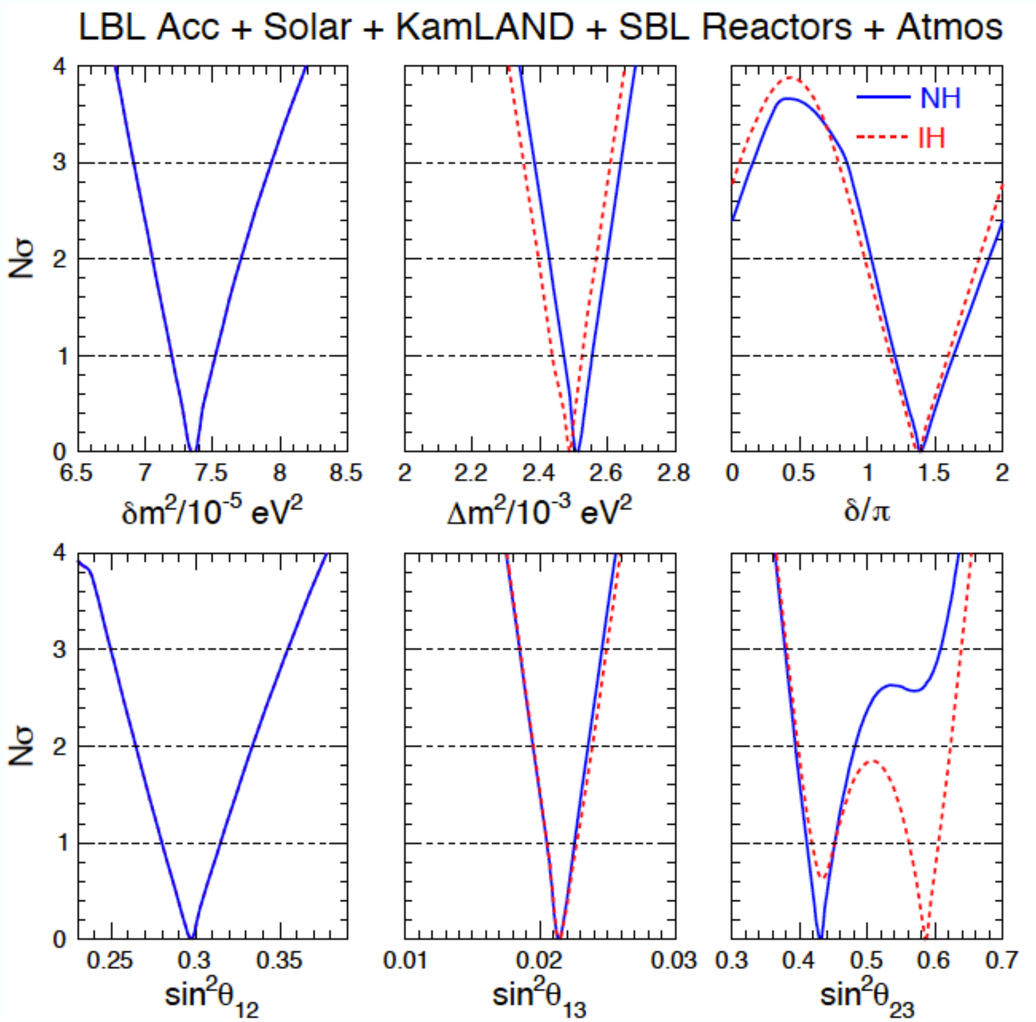
The Three-Flavor Paradigm Fits All* Data Really Well

[* modulo short-baseline anomalies]

Bounds on single oscillation parameters

(preliminary update)

[A. Marrone, Talk at Neutrino 2016]



CP phase trend:

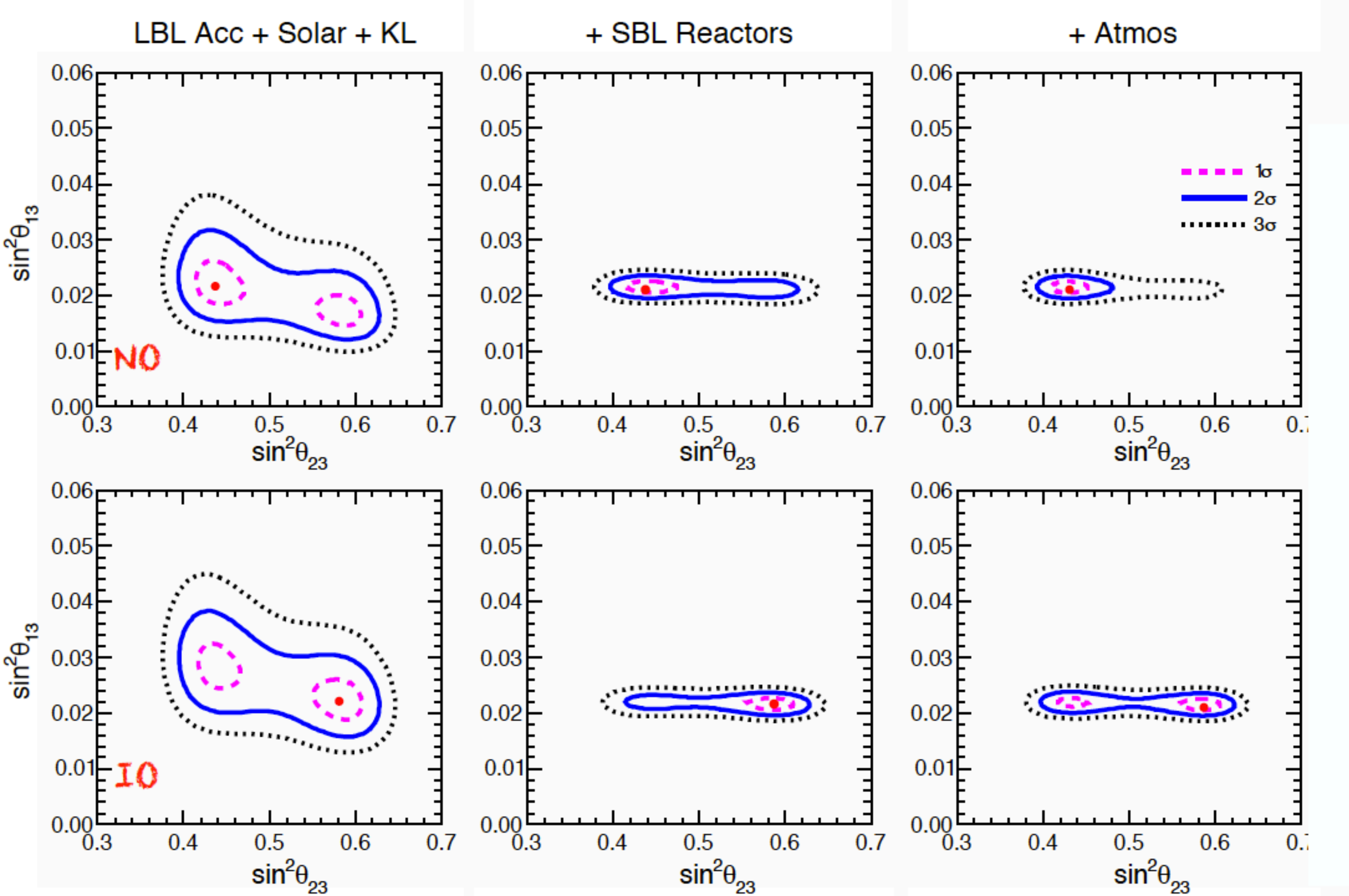
- $\delta \sim 1.4\,\pi$ at best fit
- CP-conserving cases ($\delta = 0, \pi$) disfavored at $\sim 2\sigma$ level or more
- Significant fraction of the $[0,\pi]$ range disfavored at $>3\sigma$

θ_{23} trend:

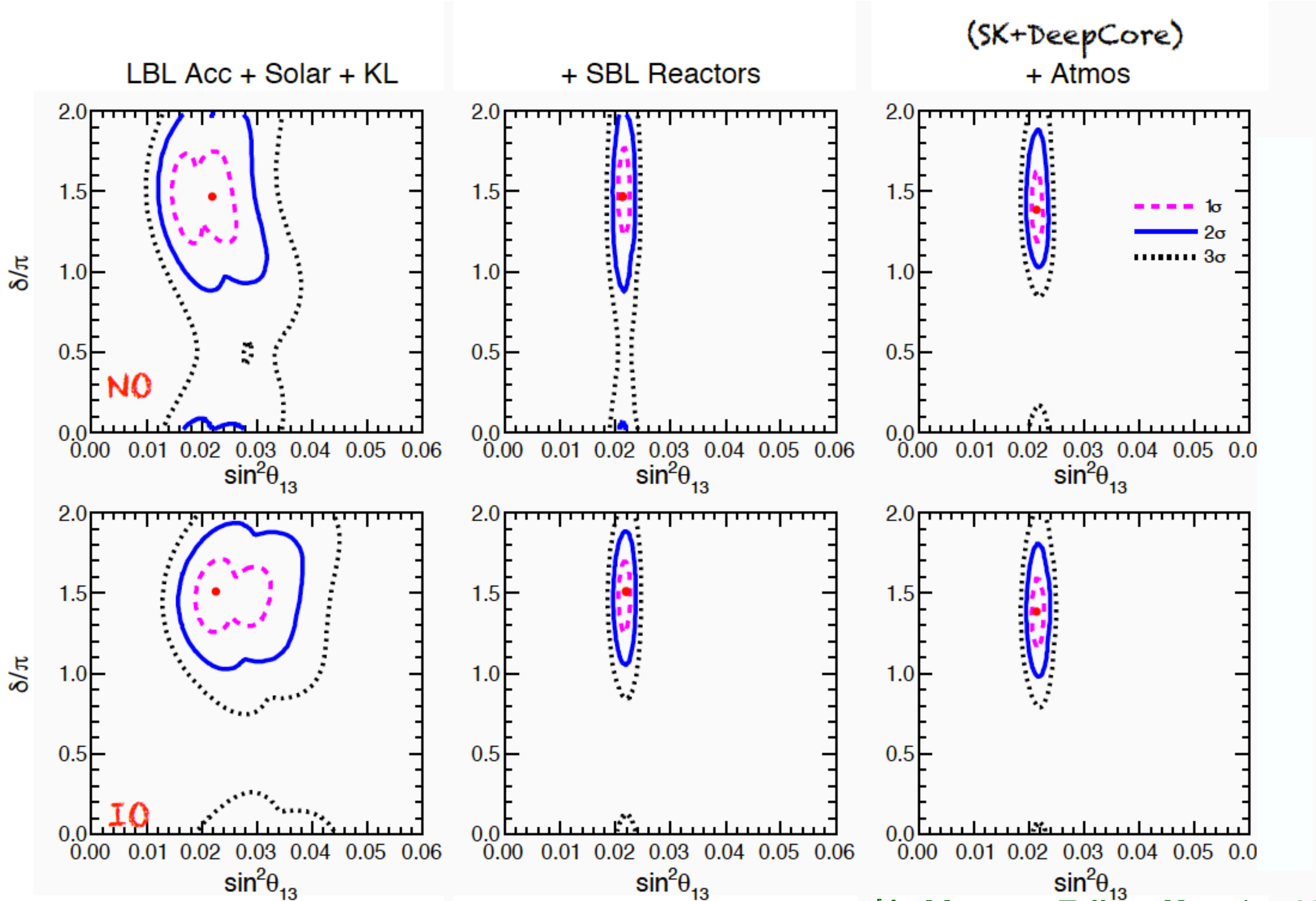
- maximal mixing disfavored at about $\sim 2\sigma$ level
- best-fit octant flips with mass ordering

$$\Delta\chi^2_{\rm IO-NO} = 3.1$$

inverted ordering slightly disfavored



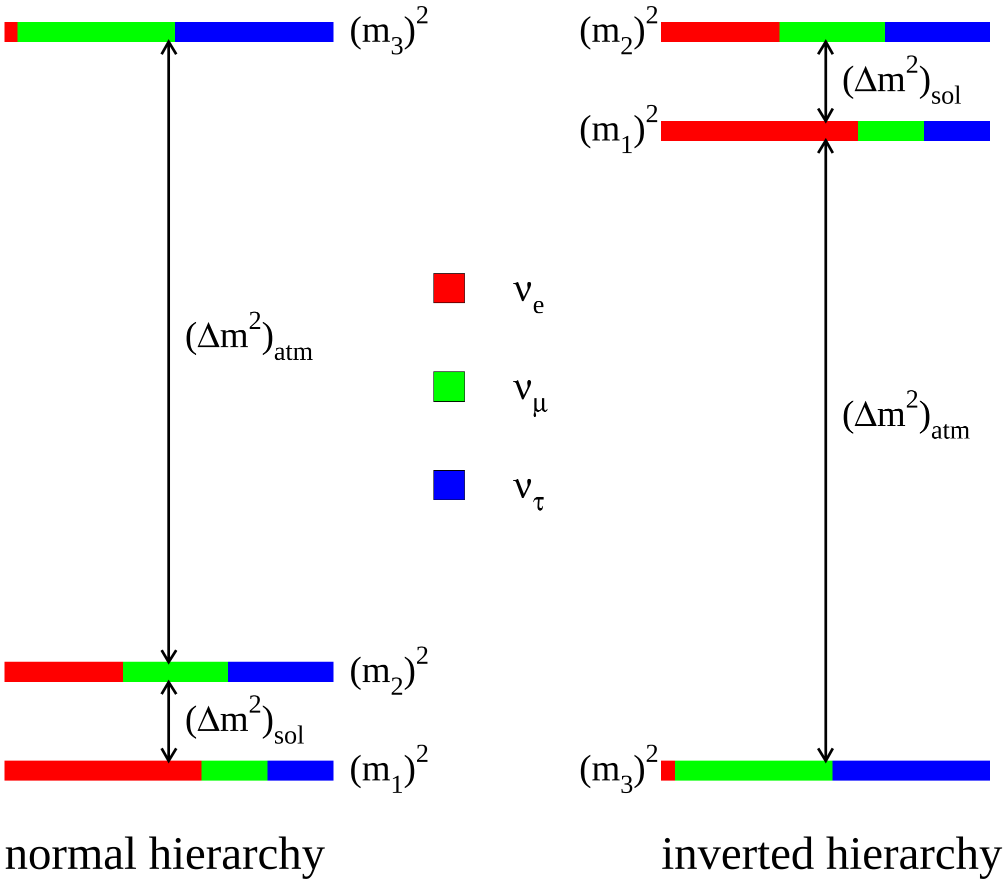
Atmospheric data do not spoil this trend but introduce some differences in the relative likelihood of the two octants in NO and IO. Octant degeneracy may show up in terms of "bumps" or "double bands" when marginalized away –> [A. Marrone, Talk at Neutrino 2016]



[A. Marrone, Talk at Neutrino 2016]

Results in the (δ, θ_{13}) plane corroborated by atmospheric data

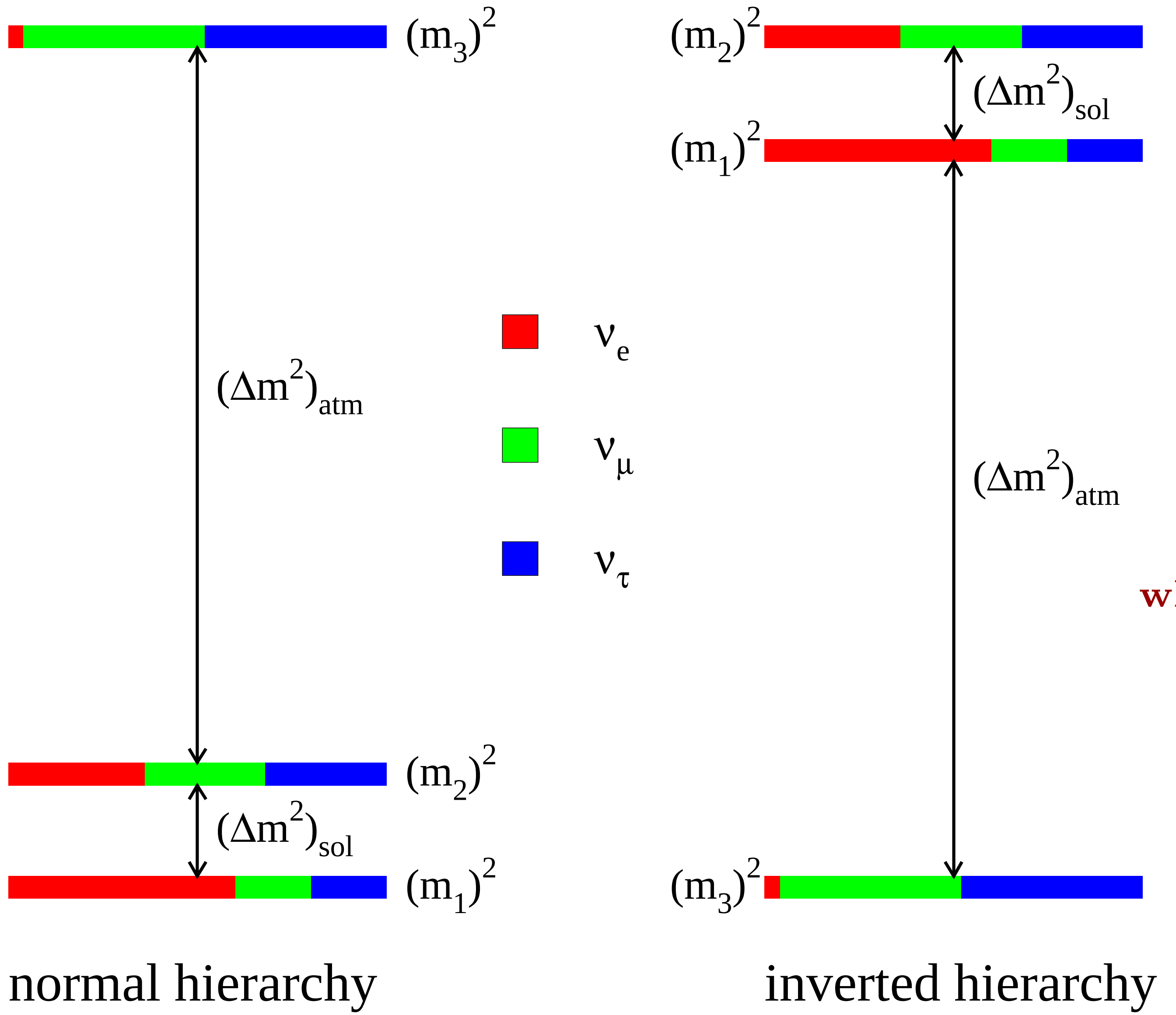
Understanding Neutrino Oscillations: Are We There Yet? [NO!]



- ~~What is the ν_e component of ν_3?~~ ($\theta_{13} \neq 0$!)
- Is CP-invariance violated in neutrino oscillations? ($\delta \neq 0, \pi$?) ['yes' hint]
- Is ν_3 mostly ν_μ or ν_τ? [$\theta_{23} \neq \pi/4$ hint]
- What is the neutrino mass hierarchy? ($\Delta m^2_{13} > 0$?) [NH weak hint]

$\Rightarrow$ All of the above can "only" be addressed with new neutrino oscillation experiments

Ultimate Goal: Not Measure Parameters but Test the Formalism (Over-Constrain Parameter Space)



The Neutrino Mass Hierarchy

which is the right picture?

Why Don't We Know the Neutrino Mass Hierarchy?

Most of the information we have regarding θ_{23} and Δm^2_{13} comes from atmospheric neutrino experiments (SuperK). Roughly speaking, they measure

$$P_{\mu\mu} = 1 - \sin^2 2\theta_{23} \sin^2\left(\frac{\Delta m^2_{13} L}{4E}\right) + \text{ subleading.}$$

It is easy to see from the expression above that the leading term is simply not sensitive to the sign of Δm^2_{13}.

On the other hand, because $|U_{e3}|^2 \sim 0.02$ and $\frac{\Delta m^2_{12}}{\Delta m^2_{13}} \sim 0.03$ are both small, we are yet to observe the subleading effects.

Determining the Mass Hierarchy via Oscillations – the large U_{e3} route

Again, necessary to probe $\nu_\mu \to \nu_e$ oscillations (or vice-versa) governed by Δm^2_{13}. This is the oscillation channel that (almost) all next-generation, accelerator-based experiments are concentrating on, including the ongoing experiments T2K and NOνA.

In vaccum

$$P_{\mu e} = \sin^2\theta_{23}\sin^2 2\theta_{13}\sin^2\left(\frac{\Delta m^2_{13}L}{4E}\right) + \text{ “subleading”},$$

so that, again, this is insensitive to the sign of Δm^2_{13} at leading order. However, in this case, matter effects may come to the rescue.

As I discussed already, neutrino oscillations get modified when these propagate in the presence of matter. Matter effects are sensitive to the neutrino mass ordering (in a way that I will describe shortly) and different for neutrinos and antineutrinos.

If $\Delta_{12} \equiv \frac{\Delta m^2_{12}}{2E}$ terms are ignored, the $\nu_\mu \to \nu_e$ oscillation probability is described, in constant matter density, by

$$P_{\mu e} \simeq P_{e\mu} \simeq \sin^2\theta_{23}\sin^2 2\theta_{13}^{\rm eff}\sin^2\left(\frac{\Delta_{13}^{\rm eff}L}{2}\right),$$

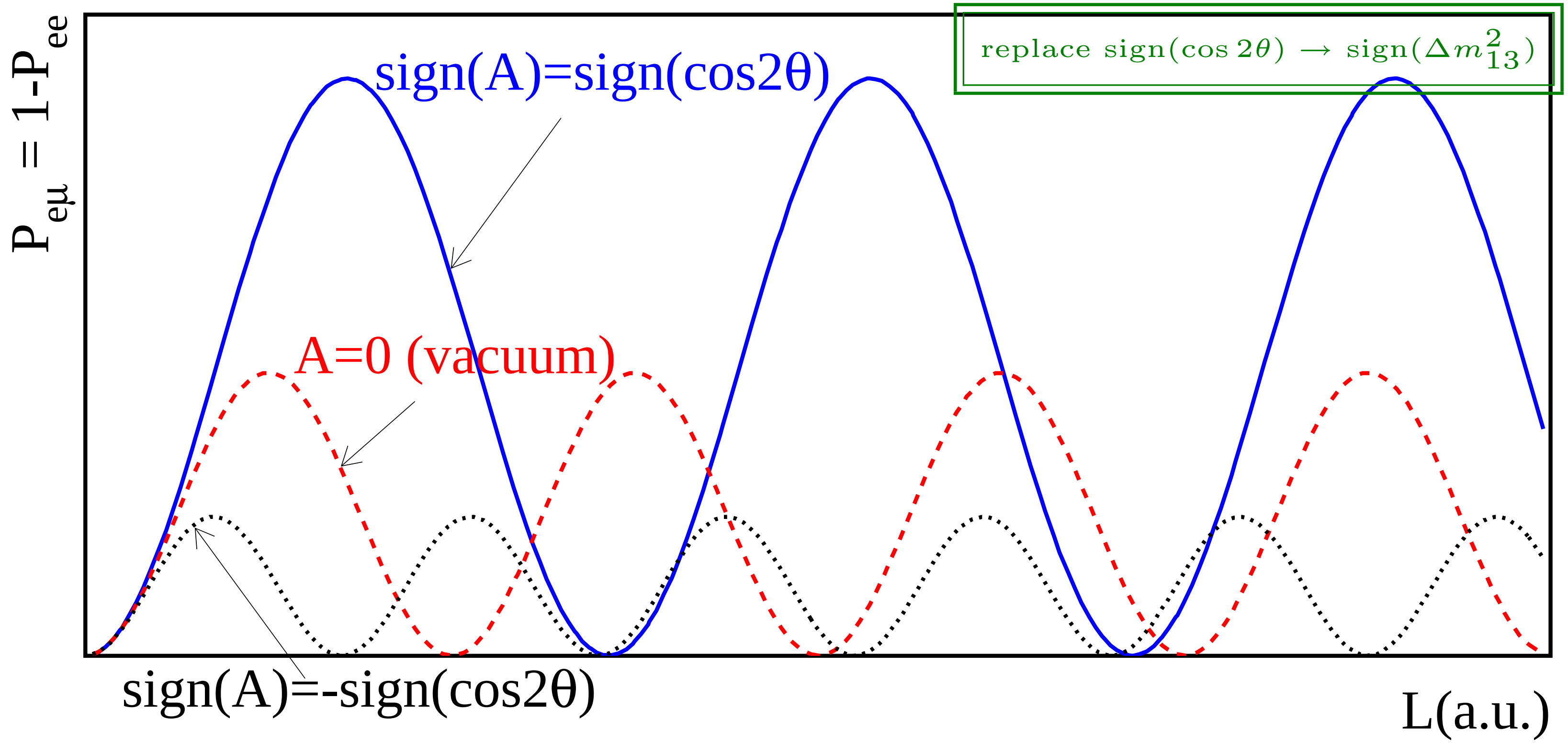
$$\sin^2 2\theta_{13}^{\rm eff} = \frac{\Delta_{13}^2\sin^2 2\theta_{13}}{(\Delta_{13}^{\rm eff})^2},$$

$$\Delta_{13}^{\rm eff} = \sqrt{(\Delta_{13}\cos 2\theta_{13} - A)^2 + \Delta_{13}^2\sin^2 2\theta_{13}},$$

$$\Delta_{13} = \frac{\Delta m^2_{13}}{2E},$$

$A \equiv \pm\sqrt{2}G_F N_e$ is the matter potential. It is positive for neutrinos and negative for antineutrinos.

$P_{\mu e}$ depends on the relative sign between Δ_{13} and A. It is different for the two different mass hierarchies, and different for neutrinos and antineutrinos.



Requirements:

- $\sin^2 2\theta_{13}$ large enough – otherwise there is nothing to see!
- $|\Delta_{13}| \sim |A|$ – matter potential must be significant but not overwhelming.
- $\Delta_{13}^{\rm eff} L$ large enough – matter effects are absent near the origin.

The "Holy Graill" of Neutrino Oscillations – CP Violation

In the old Standard Model, there is only one[a] source of CP-invariance violation:

$\Rightarrow$ The complex phase in V_{CKM}, the quark mixing matrix.

Indeed, as far as we have been able to test, all CP-invariance violating phenomena agree with the CKM paradigm:

- ϵ_K;
- ϵ'_K;
- $\sin 2\beta$;
- etc.

Recent experimental developments, however, provide strong reason to believe that this is not the case: neutrinos have mass, and leptons mix!

[a] modulo the QCD θ-parameter, which will be "willed away" henceforth.

Golden Opportunity to Understand Matter versus Antimatter?

The SM with massive Majorana neutrinos accommodates **five** irreducible CP-invariance violating phases.

- One is the phase in the CKM phase. We have measured it, it is large, and we don't understand its value. At all.
- One is θ_{QCD} term ($\theta G\tilde{G}$). We don't know its value but it is only constrained to be very small. We don't know why (there are some good ideas, however).
- Three are in the neutrino sector. One can be measured via neutrino oscillations. 50% increase on the amount of information.

We don't know much about CP-invariance violation. Is it really fair to presume that CP-invariance is generically violated in the neutrino sector solely based on the fact that it is violated in the quark sector? Why? Cautionary tale: "Mixing angles are small"

CP-invariance Violation in Neutrino Oscillations

The most promising approach to studying CP-violation in the leptonic sector seems to be to compare $P(\nu_\mu \to \nu_e)$ versus $P(\bar{\nu}_\mu \to \bar{\nu}_e)$.

The amplitude for $\nu_\mu \to \nu_e$ transitions can be written as

$$A_{\mu e} = U_{e2}^* U_{\mu 2} \left(e^{i\Delta_{12}} - 1\right) + U_{e3}^* U_{\mu 3} \left(e^{i\Delta_{13}} - 1\right)$$

where $\Delta_{1i} = \frac{\Delta m_{1i}^2 L}{2E}$, $i = 2, 3$.

The amplitude for the CP-conjugate process can be written as

$$\bar{A}_{\mu e} = U_{e2} U_{\mu 2}^* \left(e^{i\Delta_{12}} - 1\right) + U_{e3} U_{\mu 3}^* \left(e^{i\Delta_{13}} - 1\right).$$

[remember: according to unitarty, $U_{e1}U_{\mu 1}^* = -U_{e2}U_{\mu 2}^* - U_{e3}U_{\mu 3}^*$]

In general, $|A|^2 \neq |\bar{A}|^2$ (CP-invariance violated) as long as:

- Nontrivial "Weak" Phases: $\arg(U_{ei}^* U_{\mu i}) \rightarrow \delta \neq 0, \pi$;
- Nontrivial "Strong" Phases: $\Delta_{12}, \Delta_{13} \rightarrow L \neq 0$;
- Because of Unitarity, we need all $|U_{\alpha i}| \neq 0 \rightarrow$ three generations.

All of these can be satisfied, with a little luck: given that two of the three mixing angles are known to be large, **we need $|U_{e3}| \neq 0$. (✓)**

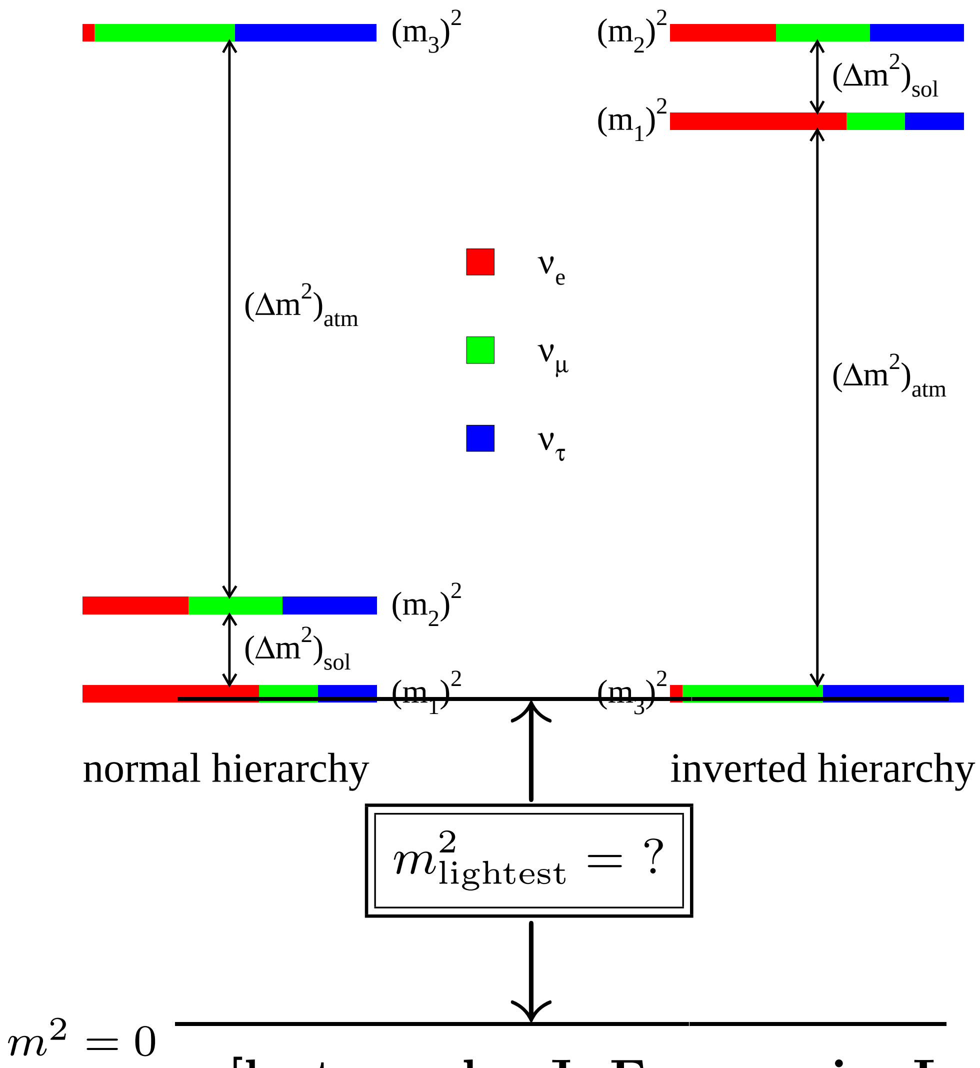
The goal of next-generation neutrino experiments is to determine the magnitude of $|U_{e3}|$. We need to know this in order to understand how to study CP-invariance violation in neutrino oscillations!

In the real world, life is much more complicated. The lack of knowledge concerning the mass hierarchy, θ_{13}, θ_{23} leads to several degeneracies.

Note that, in order to see CP-invariance violation, we **need** the "subleading" terms!

In order to ultimately measure a new source of CP-invariance violation, we will need to combine different measurements:
– oscillation of muon neutrinos and antineutrinos,
– oscillations at accelerator and reactor experiments,
– experiments with different baselines,
– etc.

4– What We Know We Don't Know (ii): How Light is the Lightest Neutrino?



So far, we've only been able to measure neutrino mass-squared differences.

The lightest neutrino mass is only poorly constrained: $m^2_{\text{lightest}} < 1\ \text{eV}^2$

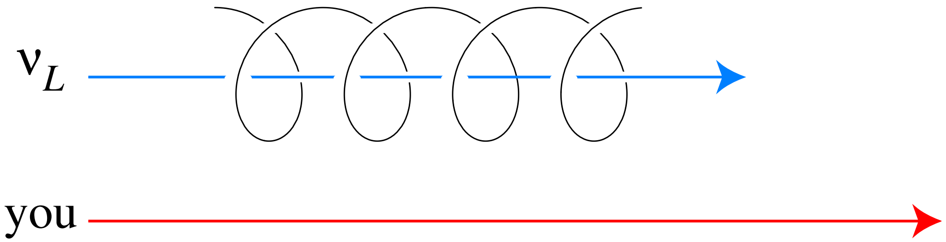
qualitatively different scenarios allowed:

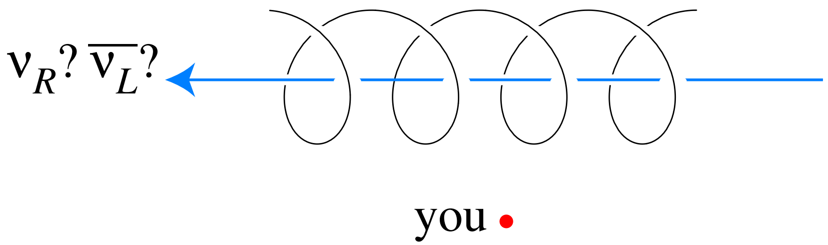
- $m^2_{\text{lightest}} \equiv 0$;
- $m^2_{\text{lightest}} \ll \Delta m^2_{12,13}$;
- $m^2_{\text{lightest}} \gg \Delta m^2_{12,13}$.

Need information outside of neutrino oscillations.

[lectures by J. Formaggio, L. Kaufman, A. Melchiorri, W. Rodejohann]

4– What We Know We Don't Know (iii) – Are Neutrinos Majorana Fermions?





A massive charged fermion (s=1/2) is described by 4 degrees of freedom:

$$(e^-_L \leftarrow \text{CPT} \rightarrow e^+_R)$$
$$\updownarrow \text{Lorentz}$$
$$(e^-_R \leftarrow \text{CPT} \rightarrow e^+_L)$$

A massive neutral fermion (s=1/2) is described by 4 or 2 degrees of freedom:

$$(\nu_L \leftarrow \text{CPT} \rightarrow \bar{\nu}_R)$$
$$\updownarrow \text{Lorentz} \qquad \text{“DIRAC”}$$
$$(\nu_R \leftarrow \text{CPT} \rightarrow \bar{\nu}_L)$$

“MAJORANA”

$$(\nu_L \leftarrow \text{CPT} \rightarrow \bar{\nu}_R)$$
$$\updownarrow \text{Lorentz}$$
$$(\bar{\nu}_R \leftarrow \text{CPT} \rightarrow \nu_L)$$

How many degrees of freedom are required to describe massive neutrinos?

Why Don't We Know the Answer (Yet)?

If neutrino masses were indeed zero, this is a nonquestion: there is no distinction between a massless Dirac and Majorana fermion.

Processes that are proportional to the Majorana nature of the neutrino vanish in the limit $m_\nu \to 0$. Since neutrinos masses are very small, the probability for these to happen is very, very small: $A \propto m_\nu/E$.

The "smoking gun" signature is the observation of LEPTON NUMBER violation. This is easy to understand: Majorana neutrinos are their own antiparticles and, therefore, cannot carry any quantum numbers — including lepton number.

The deepest probes are searches for Neutrinoless Double-Beta Decay. These will be discussed by L. Kaufman and W. Rodejohann.

Weak Interactions are Purely Left-Handed (Chirality):

For example, in the scattering process $e^- + X \to \nu_e + X$, the electron neutrino is, in a reference frame where $m \ll E$,

$$|\nu_e\rangle \sim |L\rangle + \left(\frac{m}{E}\right)|R\rangle.$$

If the neutrino is a Majorana fermion, $|R\rangle$ behaves mostly like a "$\bar{\nu}_e$," (and $|L\rangle$ mostly like a "ν_e,") such that the following process could happen:

$$e^- + X \to \nu_e + X, \quad \text{followed by} \quad \nu_e + X \to e^+ + X, \qquad P \simeq \left(\frac{m}{E}\right)^2$$

Lepton number can be violated by 2 units with small probability. Typical numbers: $P \simeq (0.1~\text{eV}/100~\text{MeV})^2 = 10^{-18}$. VERY Challenging!

How many new CP-violating parameters in the neutrino sector?

If the neutrinos are Majorana fermions, there are more physical observables in the leptonic mixing matrix.

Remember the parameter counting in the quark sector:

9 $(3 \times 3$ unitary matrix)

$\underline{-5\quad}$ (relative phase rotation among six quark fields)

4 (3 mixing angles and 1 CP-odd phase).

If the neutrinos are Majorana fermions, the parameter counting is quite different: there are no right-handed neutrino fields to "absorb" CP-odd phases:

$$\begin{array}{rl} 9 & (3 \times 3 \text{ unitary matrix}) \\ \underline{-3\quad} & (\text{three right-handed charged lepton fields}) \\ 6 & (3 \text{ mixing angles and } 3 \text{ CP-odd phases}). \end{array}$$

There is CP-invariance violating parameters even in the 2 family case: $4 - 2 = 2$, one mixing angle, one CP-odd phase.

$$\mathcal{L} \supset \bar{e}_L U W^{\mu}\gamma_{\mu}\nu_L - \bar{e}_L(M_e)e_R - \overline{\nu_L^c}(M_\nu)\nu_L + H.c.$$

Write $U = E^{-i\xi/2}U'E^{i\alpha/2}$, where $E^{i\beta/2} \equiv \mathrm{diag}(e^{i\beta_1/2}, e^{i\beta_2/2}, e^{i\beta_3/2})$, $\beta = \alpha, \xi$

$$\mathcal{L} \supset \bar{e}_L U' W^{\mu}\gamma_{\mu}\nu_L - \bar{e}_L E^{i\xi/2}(M_e)e_R - \overline{\nu_L^c}(M_\nu)E^{-i\alpha}\nu_L + H.c.$$

ξ phases can be "absorbed" by e_R,

α phases cannot go away!

on the other hand

Dirac Case:

$$\mathcal{L} \supset \bar{e}_L U W^{\mu}\gamma_{\mu}\nu_L - \bar{e}_L(M_e)e_R - \bar{\nu}_R(M_\nu)\nu_L + H.c.$$

$$\mathcal{L} \supset \bar{e}_L U' W^{\mu}\gamma_{\mu}\nu_L - \bar{e}_L E^{i\xi/2}(M_e)e_R - \bar{\nu}_R(M_\nu)E^{-i\alpha/2}\nu_L + H.c.$$

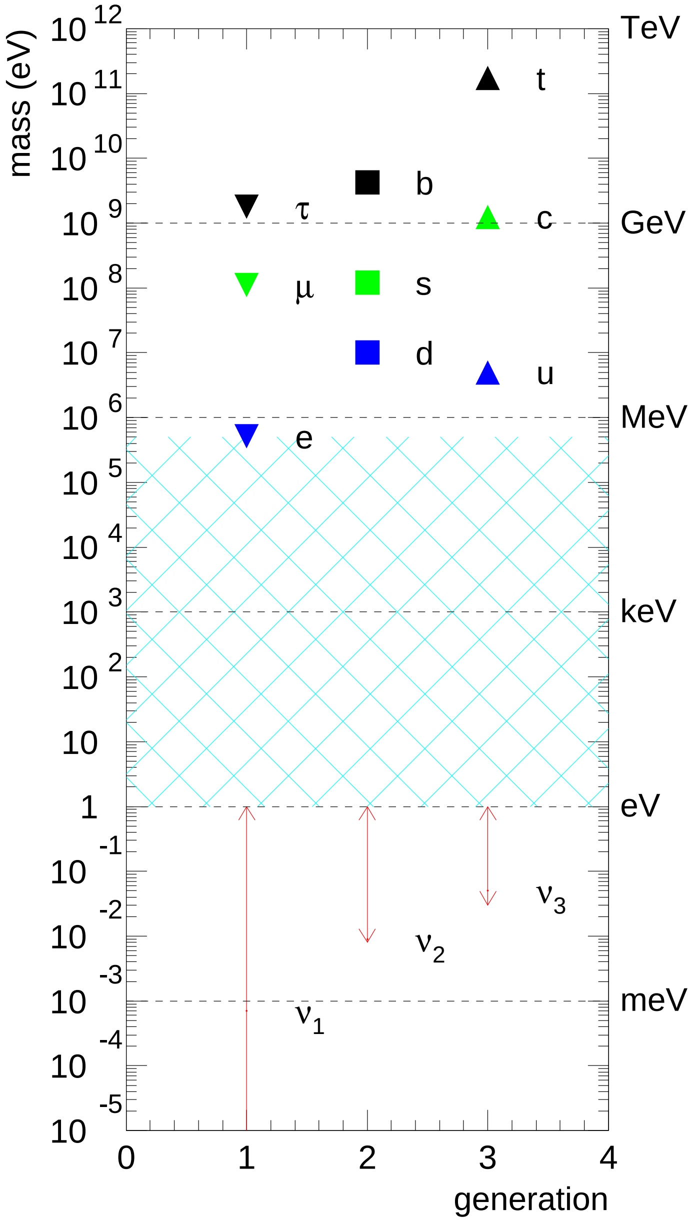
ξ phases can be "absorbed" by e_R, α phases can be "absorbed" by ν_R,

$$V_{MNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{e\tau 2} & U_{\tau 3} \end{pmatrix}' \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & e^{i\alpha_3/2} \end{pmatrix}.$$

It is easy to see that the Majorana phases never show up in neutrino oscillations ($A \propto U_{\alpha i}U^*_{\beta_i}$).

Furthermore, they only manifest themselves in phenomena that vanish in the limit $m_i \to 0$ – after all they are only physical if we "know" that lepton number is broken.

$$A(\alpha_i) \propto m_i/E \quad \to \quad \text{tiny!}$$



NEUTRINOS HAVE MASS

albeit very tiny ones…

SO WHAT?

Only* "Palpable" Evidence of Physics Beyond the Standard Model

The SM we all learned in school predicts that neutrinos are strictly massless. Hence, massive neutrinos imply that the the SM is incomplete and needs to be replaced/modified.

Furthermore, the SM has to be replaced by something qualitatively different.

---

* There is only a handful of questions our model for fundamental physics cannot explain (these are personal. Feel free to complain).

- What is the physics behind electroweak symmetry breaking? (Higgs ✓).
- What is the dark matter? (not in SM).
- Why is there more matter than antimatter? (Not in SM).
- Why does the Universe appear to be accelerating? Why does it appear that the Universe underwent rapid acceleration in the past? (not in SM).

Standard Model in One Slide, No Equations

The SM is a quantum field theory with the following defining characteristics:

- Gauge Group $(SU(3)_c \times SU(2)_{\rm L} \times U(1)_{\rm Y})$;
- Particle Content (fermions: Q, u, d, L, e, scalars: H).

Once this is specified, the SM is unambiguously determined:

- Most General Renormalizable Lagrangian;
- Measure All Free Parameters, and You Are Done! (after several decades of hard experimental work…)

If you follow these rules, neutrinos have no mass. Something has to give.

What is the New Standard Model? [νSM]

The short answer is – WE DON'T KNOW. Not enough available info!

$$\Updownarrow$$

Equivalently, there are several completely different ways of addressing neutrino masses. The key issue is to understand what else the νSM candidates can do. [are they falsifiable?, are they "simple"?, do they address other outstanding problems in physics?, etc]

We need more experimental input, and it looks like it may be coming in the near/intermediate future!

Neutrino Masses, EWSB, and a New Mass Scale of Nature

The LHC has revealed that the minimum SM prescription for electroweak symmetry breaking — the one Higgs double model — is at least approximately correct. What does that have to do with neutrinos?

The tiny neutrino masses point to three different possibilities.

1. Neutrinos talk to the Higgs boson very, very **weakly** (Dirac neutrinos);
2. Neutrinos talk to a **different Higgs** boson – there is a new source of electroweak symmetry breaking! (Majorana neutrinos);
3. Neutrino masses are small because there is **another source of mass** out there — a new energy scale indirectly responsible for the tiny neutrino masses, a la the seesaw mechanism (Majorana neutrinos).

Searches for $0\nu\beta\beta$ help tell (1) from (2) and (3), the LHC, charged-lepton flavor violation, *et al* may provide more information.

Understanding Fermion Mixing

The other puzzling phenomenon uncovered by the neutrino data is the fact that Neutrino Mixing is Strange. What does this mean?

It means that lepton mixing is very different from quark mixing:

$$V_{MNS} \sim \begin{pmatrix} 0.8 & 0.5 & \mathbf{0.2} \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix} \qquad V_{CKM} \sim \begin{pmatrix} 1 & 0.2 & 0.001 \\ 0.2 & 1 & 0.01 \\ 0.001 & 0.01 & 1 \end{pmatrix}$$

$[|(V_{MNS})_{e3}| < 0.2]$

$\boxed{\text{WHY?}}$

They certainly look VERY different, but which one would you label as "strange"?

Piecing the Neutrino Mass Puzzle

Understanding the origin of neutrino masses and exploring the new physics in the lepton sector will require unique **theoretical** and **experimental** efforts, including …

- understanding the fate of lepton-number. Neutrinoless double beta decay!
- a comprehensive long baseline neutrino program, towards precision oscillation physics.
- other probes of neutrino properties, including neutrino scattering.
- precision studies of charged-lepton properties ($g-2$, edm), and searches for rare processes ($\mu \to e$-conversion the best bet at the moment).
- collider experiments. The LHC and beyond may end up revealing the new physics behind small neutrino masses.
- cosmic surveys. Neutrino properties affect, in a significant way, the history of the universe. Will we learn about neutrinos from cosmology, or about cosmology from neutrinos?
- searches for baryon-number violating processes.

CONCLUSIONS

The venerable Standard Model has finally sprung a leak – neutrinos are not massless!

1. we have a very successful parametrization of the neutrino sector, and we have identified what we know we don't know.
2. neutrino masses are very small – we don't know why, but we think it means something important.
3. lepton mixing is very different from quark mixing – we don't know why, but we think it means something important.
4. we need a minimal νSM Lagrangian. In order to decide which one is "correct" (required in order to attack 2. and 3. above) we must uncover the faith of baryon number minus lepton number ($0\nu\beta\beta$ is the best [only?] bet).

5. We need more experimental input – and more seems to be on the way (this is a truly data driven field right now). We only started to figure out what is going on.

6. The fact that neutrinos have mass may be intimately connected to the fact that there are more baryons than antibaryons in the Universe. How do we test whether this is correct?

7. There is plenty of room for surprises, as neutrinos are very narrow but deep probes of all sorts of physical phenomena. Remember that neutrino oscillations are "quantum interference devices" – potentially very sensitive to whatever else may be out there.